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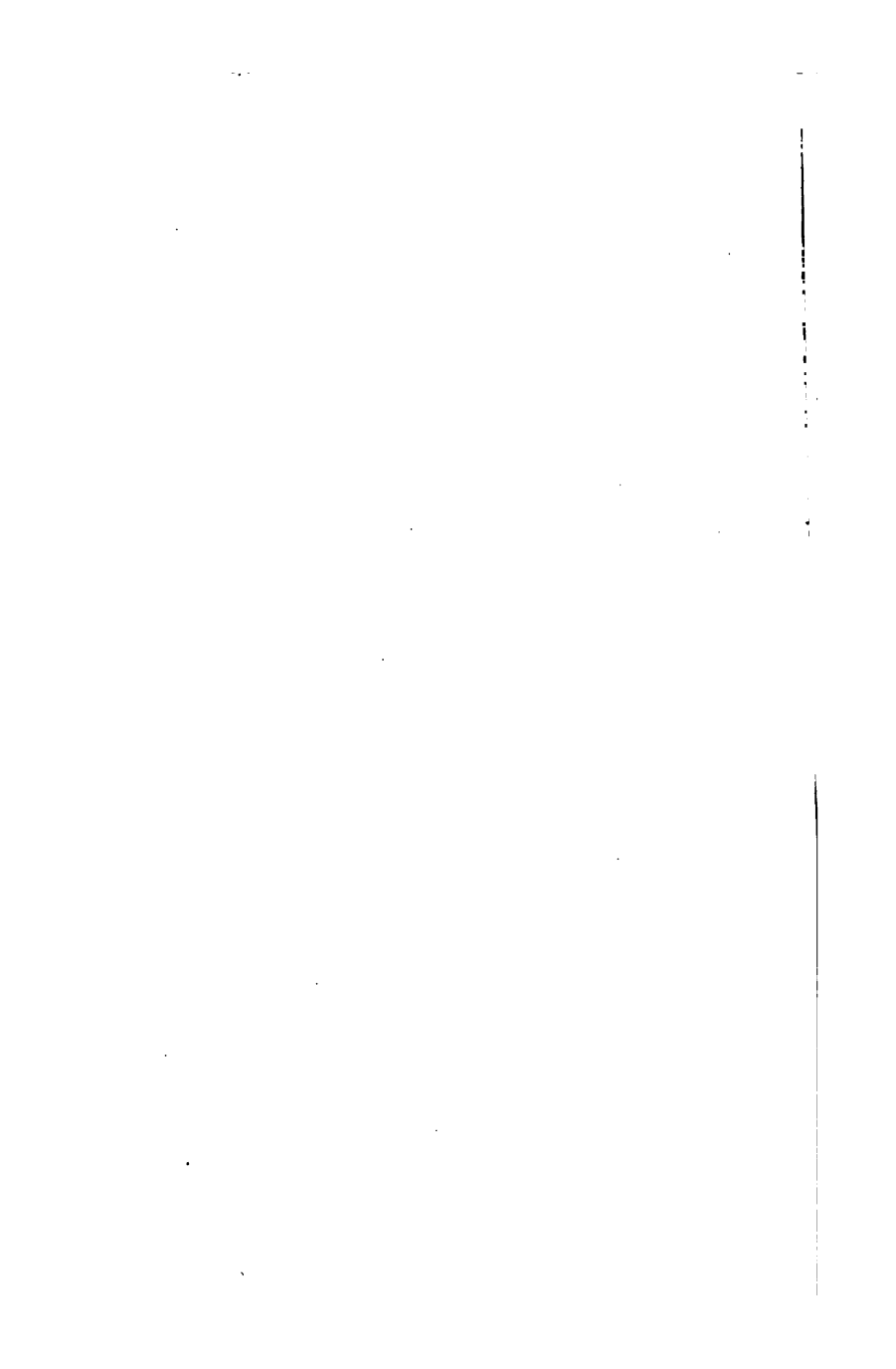
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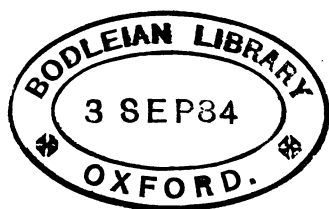
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PREFACE.

I HAVE arranged in this book a series of Examples in Elementary Algebra co-extensive with my Treatise on that subject. The Examples are progressive and easy. They have been selected chiefly from papers set during the last three years in University and College Examinations.

The Exercises are arranged on the following plan :

Part I. conducts the Student by gradual steps as far as Geometrical Progression : each exercise having the limit of its extent specified in the heading by a reference to the Chapters of my **ELEMENTARY ALGEBRA**.

Part II. contains papers of greater length and somewhat more difficulty than those in Part I. No question in these papers implies a knowledge of any part of Algebra beyond Geometrical Progression, but at the end of each exercise one piece of *bookwork* is given.

Part III. takes in the whole of the subject, so far as I have written on it in my Treatise, especial pro-

minence being given to that portion of the work which follows the Chapter on Geometrical Progression.

The questions on bookwork in Parts II. and III. follow the order in which the matters to which they refer are given in my Treatise.

I have the pleasure of expressing my thanks to my friend Mr J. HENRY, Mathematical Master in the School, H. M. Dockyard, Sheerness, for valuable assistance in testing the greater part of these Examples.

J. HAMBLIN SMITH.

Cambridge, 1870.

PART I.

EXERCISE—I. (CHAPS. I. AND II.)

1. Add together $3x+4y-5z$, $x+3y+7z$, and $2x-7y-2z$.
2. From $7a-2b+3c$ take $2a-5b-8c$.
3. Simplify $17x-4y-[3y+2x-\{5y-6x-(2y-x)\}]$.
4. Multiply $x^2-3xy+y^2$ by $x^2+3xy+y^2$.
5. Multiply $a-b+c-d$ by $a+b-c+d$.
6. Multiply $3x^3-5x^2+4x-7$ by $2x^2-3x+5$.

EXERCISE—II. (CHAPS. I. TO III.)

1. Add together $-5x^2+7x^2y-3xy^2$, $4xy^2-9x^2y+2x^3$, and $17x^2y+6x^3-2xy^2$.
2. From $-a^3+3b^3-4c^3+5abc$ take $7abc+3c^3-2b^3-8a^3$.
3. Simplify $5x^2+3xy-2y^2-(3x^2-5xy-7y^2)-(x^2-2xy+y^2)$ and find the value of the result when $x=2$ and $y=3$.
4. Multiply $5a^2-7ab-3b^2$ by $2a^2+3ab+9b^2$.
5. Shew that
$$(x+5)(3x-1)-(2x+7)(5x-2)-(9-7x^2)=-17x.$$
6. Find the value of $(x-y)^2+(y-z)^2-(x-z)^2$, when $x=2$, $y=3$, $z=4$.
7. Write down the squares of $x-9$, $3x-2y$, $2x+3y-x$ and $4a^2-5b^2$.

EXERCISE—III. (CHAPS. II. TO IV.)

1. Multiply $x + y - xy - 1$ by $x + y + xy + 1$.
2. Find the coefficient of x in the expansion of $(x-3)(2x-4)(3x+7)$.
3. Divide $x^3 - xy - 6y^2 - 4x + 12y$ by $x - 3y$.
4. Divide $1 - x + x^2 - x^3$ by $1 - x$.
5. Form the squares of $m - n + p + r$ and $a^2 - b^2 + c^2$.
6. Expand $(m - n)^3$ and $(x - y + z)^2$.

EXERCISE—IV. (CHAPS. II. TO IV.)

1. Multiply together $a^2 - a + 1$, $a^2 + a + 1$ and $a^4 - a^2 + 1$.
2. Form the square of $x^3 + 2x^2 - x + 2$.
3. Form the cube of $a^2 - 2a + 1$.
4. Divide $a^4 - ab^3 - ac^3 - 2a^2b + 2b^4 + 2bc^2 + 3a^3c - 3b^2c - 3c^4$ by $a + 3c - 2b$.
5. Find the coefficient of x in the expansion of $(x-a)(x-2b)(x-3c)$.
6. Simplify $(a+b)^2 \cdot (a-b)^2 - (a^2 + b^2)^2$.

EXERCISE—V. (CHAPS. I. TO V.)

1. Simplify $17x - (3x - 2y) - \{5y + 3x - (2y - x)\}$.
2. Multiply $4a^2 - 3ab + 7b^2$ by $3a^2 - 2ab - 9b^2$, and prove the result by division.
3. Divide $8a^3 - 64b^3$ by $2a - 4b$, and prove the result by multiplication.
4. Divide $m^6 - 2m^3 + 1$ by $m^2 - 2m + 1$.

5. Find the coefficient of x in the following expansions :

$$(1) (x-2)(x+3)(x-5). \quad (2) (x+a)(x-b)(x+c).$$

6. Express in their simplest factors :

$$(a) x^2 + 15x + 26. \quad (\beta) x^2 - (m-n)x - mn.$$

$$(\gamma) 3m^2x^2 - 3mnxy - 18n^2y^2. \quad (\delta) m^2 - (p-q)^2.$$

EXERCISE—VI. (CHAPS. III. TO V.)

1. Form the square of $a+b-x-y$.

2. Divide $a^3 - b^3 - c^3 - 3abc$ by $a-b-c$.

3. Divide $a^4 - (x^2 - y - z)a^2 - (y - z)ax + yz$ by $a^2 - ax + z$.

4. Express in the simplest factors :

$$(1) a^6 + b^6. \quad (2) a^6 - b^6. \quad (3) 4b^2x^3 - 36bx.$$

5. Find the value of $(a-b)^2 + (b-c)^2 + (c-a)^2$ when $a=4$, $b=3$, $c=2$.

6. Express algebraically :

(1) The excess of a over the sum of b and c .

(2) The number of pence in the sum of x pounds and y shillings.

EXERCISE—VII. (CHAPS. IV. AND V.)

1. Divide $a^6 - a^2b^4 - a^4b^2 + b^6$ by $a - b$.

2. Divide $(m^2 + 2m)^2 - 6(m^2 + 2m) - 27$ by $m^2 + 2m + 3$.

3. Resolve into factors :

$$(1) ac - bc + ad - bd. \quad (2) a^2 - 19a + 84.$$

$$(3) x^2 - 7x - 60. \quad (4) a^2 - (m-n)^2.$$

4. Express algebraically the number which exceeds twice x by 7.
5. By how much does $2x-3$ exceed $3x-17$?
6. Find the value of $a^3-3a^2b-3ab^2+b^3$ when $a=2$ and $b=7$.
7. Find the value of $(a-b)(b-c)-(c-d)(d-a)+(a-c)(b-d)$ when $a=d$.

EXERCISE—VIII. (CHAPS. IV. TO VI.)

1. Divide $a^3b-ab^3+a^3c+ac^3-b^2c+bc^2+abc$ by $a-b+c$.
2. If $a=1$, $b=c=2$, $d=3$, find the value of $ab^2d-a^2cd^2+bcd-b^3(3ad-2b^2+4c)-b^3(4cd-3ac^2)$.
3. Divide $a^3+b^3-c^3+3abc$ by $a+b-c$.
4. Resolve into factors:
 - (1) $a^2+9a-36$.
 - (2) x^3+22x^2+121 .
 - (3) $y^3-12xz-4x^2-9z^2$.
5. Solve the equations:
 - (1) $7x-3=15x-19$.
 - (2) $6x-(3-2x)=5x-(8x-30)$.
 - (3) $(5-3x)(7-2x)=(11-6x)(3-x)$.
6. By how many inches do x yards exceed y feet?

EXERCISE—IX. (CHAPS. II. TO VII.)

1. Multiply $x^2+2ax+a^2$ by $x^2-2ax+a^2$.
2. Divide $x^3+x^4y^4+y^8$ by x^2+xy+y^2 .
3. Prove that $(2a-b)^2+(2b-c)^2+(2c-a)^2+2(2a-b)(2b-c)+2(2b-c)(2c-a)+2(2c-a)(2a-b)-(a+b+c)^2=0$.
4. Which of the expressions c^3+d^3 , c^3-d^3 , c^3+d^3 , c^3+d^3 are divisible by $c+d$?

5. Solve the equation :

$$(x-3)(x-5)+(x-6)(x-7)=(2x-9)(x-4)-23.$$

6. Forty-two years hence a boy will be seven times as old as he was six years since : how old is he ?

EXERCISE—X. (CHAPS. II. TO VII.)

1. Multiply $x^2+3ax-a^2$ by $x^2-3ax+a^2$.

2. Divide $x^3+x^2y^4+y^8$ by x^2-xy+y^2 .

3. Prove that $(3a-b)^2+(3b-c)^2+(3c-a)^2+2(3a-b)(3b-c)+2(3b-c)(3c-a)+2(3c-a)(3a-b)-4(a+b+c)^2=0$.

4. Which of the expressions c^3-d^3 , c^4+d^4 , c^5-d^5 , c^6+d^6 are divisible by $c-d$?

5. Solve the equation:

$$(x-3)(5-x)+(x-6)(x+7)-2(3x-11)+22=0.$$

6. I have £1. 9s. 4d. in half-crowns, florins, and pence. I have twice as many half-crowns as I have florins, and twice as many pence as half-crowns. How many have I of each sort ?

EXERCISE—XI. (CHAPS. II. TO VII.)

1. Simplify $x^2-[(x-y)^2-\{(x-y-z)^2-(x-z)^2\}]$.

2. Multiply together the five factors :

$$(a-2)(a-1)a.(a+1)(a+2),$$

and divide the product by a^2-a-2 .

3. Divide $mqx^4+(nq+mr)x^3+nrx^2+pqx+pr$ by $qx+r$, and test your result by putting $x=10$, $q=r=1$, and $m=n=p=2$.

4. Resolve into factors:

$$(1) 3x^2-9x-12.$$

$$(2) 2x^3-16x^2+24x^2.$$

$$(3) ab-ac-b^2+bc.$$

5. Solve the equations :

$$(1) (x+3)(3x-2)-(x+5)(3x-7)=3x+1.$$

$$(2) (x-1)(x+2)+(x-3)(x-4)=x(2x-4).$$

6. Find three consecutive numbers whose sum is 33.

EXERCISE—XII. (CHAPS. II. TO VII.)

1. Multiply $x^2 + (a-b)x - ab$ by $x^2 + (c-d)x - cd$.

2. Prove that

$$(7a-16)(7a-17)^2-7a+16=(7a-16)^2(7a-18).$$

3. Prove that the sum of the squares of the sum and difference of any two numbers is double the sum of their squares.

4. Resolve into factors :

$$(1) (a+b)^2-(c+d)^2+(a+c)^2-(b+d)^2.$$

$$(2) x^3-y^3-z^3+2yz+x+y-z.$$

5. A debt of £3. 14s. was paid in florins and half-crowns, and one more of the former was paid than of the latter. How many florins were paid ?

6. If a, b, c be three consecutive numbers, c being the greatest, shew that the difference between the squares of a and b is less by 3 than $2c$.

EXERCISE—XIII. (CHAPS. V. TO VIII.)

1. Express in factors :

$$x^3-17x-60; 8a^3-27b^3c^3; 3a^3+3ab-90b^2.$$

2. Solve the equations :

$$(1) (x-2)^2+(x-3)^2=2(x+1)(x+3)-101.$$

$$(2) (3x^2-2x+1)(2x-1)=6x^3-7x^2+9x-6.$$

3. A man buys a horse and its harness for £24 and the horse cost seven times as much as the harness. Find the cost of the horse.

4. A has £80 and B has £32 and each loses a certain sum. Then A has five times as much as B . What is the sum lost by each?

5. Find the H. C. F. of a^3b^2c , a^2bc^2 , and abc^4 ; and also of $5x^2yz^2$, $12xy^2z$, and $20x^2y^2z^2$.

6. Find the H. C. F. of $x^2 - 12x + 27$ and $x^2 - 19x + 48$.

EXERCISE—XIV. (CHAPS. V. TO VIII.)

1. Factorize $a^5 - b^5$; $3a^7b - 3a^4b^4$, and $a^2 + b^3 - 2ab + a - b$.

2. Solve the equations:

$$(1) \quad 7x - \{(3x - 2) - (5x + 3)\} = 27 - 2[3x - (x + 2)].$$

$$(2) \quad (x + 3)(x + 4)(x + 5) = x(x + 7)^2 - 2(x - 12)^2.$$

3. Divide 25 into two such parts that six times the greater exceeds twice the less by 70.

4. Divide a line of 120 feet into three parts such that the first is twice as long as the second, and the second three times as long as the third.

5. Find the H. C. F. of $6x^2(u - x)$, $8x^3(a^2 - x^2)$ and $24a^4(a^3 - x^3)$.

6. Find the H. C. F. of $x^3 + x^2 + x + 1$ and $x^3 + 3x^2 + 3x + 1$.

EXERCISE—XV. (CHAPS. VII. TO IX.)

1. Divide 48 into four such parts that the first increased by 3, the second diminished by 3, the third multiplied by 3, and the fourth divided by 3, shall all give the same result.

2. A and B have equal sums of money. A gains £150 and B loses £75, and then A has twice as much as B . How much had each at first?

3. Find the H. C. F. of $x^3 - 8x + 3$ and $x^3 + 3x^2 + x + 3$.

4. Find the H. C. F. of $15x^2 + 11x - 14$ and $3x^2 + x^2 - 5x + 2$.
5. Reduce to its lowest terms $\frac{2x^2 - 7x - 2}{2x^2 - x - 6}$.
6. Reduce to its lowest terms $\frac{4x^3 - xy - 3y^2}{12x^3 - 12x^2y + 4xy^2 - 4y^3}$.

EXERCISE—XVI. (CHAPS. VIII. AND IX.)

1. Find the H. C. F. of
 $2x^3 + 9x^2 + 4x - 15$ and $4x^3 + 8x^2 + 3x + 20$.
2. Find the H. C. F. of
 $6x^2 - 11x^2y + 18xy^2 - 7y^3$ and $14x^2 - 15xy + 4y^2$.
3. Reduce to its lowest terms $\frac{3a^3 - 22a - 15}{5a^4 - 17a^3 + 18a}$.
4. Reduce to its lowest terms $\frac{7m^3 - 11m^2 + 15m + 9}{m^4 + 2m^2 + 9}$.
5. Find the H. C. F. of
 $x^2 - 17x - 38$, $x^2 + 11x - 570$, and $x^2 - 21x^2 + 45x - 133$.
6. Simplify

$$\frac{(x-y)^2 - z^2}{(x+y)^2 - z^2} \times \frac{z^2 - (x+y)^2}{(x-y)^2 - x^2} \times \frac{x^2 - (y-z)^2}{x^2 - (y+z)^2} \times \frac{x+y+z}{x-y+z}.$$

EXERCISE—XVII. (CHAPS. VIII. TO XI.)

1. Find the L. C. M. of $12a^3x^5$, $18a^4x^2y^3$, and $72ay^4$.
2. Find the L. C. M. of $7(a-b)$, $14(a^2-b^2)$, and $21(a^3-b^3)$.
3. Find the H. C. F. of
 $6x^4 - x^2y^2 - 12y^4$ and $9x^5 + 12x^2y^2 - 6x^2y^3 - 8y^5$.
4. Reduce to its lowest terms $\frac{a^4 - a^3 - a + 1}{a^4 + a^3 - a - 1}$.

5. Simplify $\frac{1}{x^2-y^2} + \frac{1}{(x+y)^2} - \frac{1}{(x-y)^2}$.
6. Simplify $\frac{a^3-b^3}{a^3-b^3} \div \frac{a^3+2ab+b^3}{a^3+ab+b^3}$.
7. Simplify $\frac{1}{x^2-7x+12} - \frac{1}{x^2+x-12}$.

EXERCISE—XVIII. (CHAPS. II. TO XI.)

1. Find the coefficient of x in the expansion of

$$(3x-a)(4x-b)(5x-c).$$

2. Divide $x^4-(a+c)x^3+(b+ac)x^2-bcx$ by $x-c$.

3. Find the H. C. F. of

$$2m^3-3m^2-2m+3 \text{ and } 3m^4+2m^3-2m^2-2m-1.$$

4. Factorize

$$5x^2+60xy+175y^2; m^6-64; a^2+2ab+b^2+a+b.$$

5. Simplify $\frac{x^2-7x+12}{x^2-x} \times \frac{x^2-1}{x^2-4x} \div \frac{x^2+x+1}{x^2}$.

6. Simplify $\frac{2a}{a-b} + \frac{3a}{a+b} - \frac{5a^2-7ab}{a^2+b^2}$.

7. Simplify $\frac{x-1}{x+1} + \frac{1+x}{1-x} + \frac{4x^2}{x^2-1}$.

EXERCISE—XIX. (CHAPS. IX. TO XII.)

1. Reduce to its lowest terms $\frac{3x^3+x^2+x-2}{x^3-1}$.
2. Simplify $\frac{(a-b)^4}{m^3+n^3} \div \left(\frac{a^3-2ab+b^3}{m+n} \times \frac{a^2-b^2}{m^2-mn+n^2} \right)$.
3. Simplify $\frac{5+3x}{2-x} - \frac{5-3x}{2+x} + \frac{48-2x}{x^2-4}$.

4. Find the L. C. M. of

$$x^2 + 23x - 50, \quad x^2 + 5x - 14, \quad \text{and} \quad x^2 + 28x^2 + 47x - 700.$$

5. Simplify $\frac{2}{(x-2)(x-5)} + \frac{3}{(x-3)(5-x)} + \frac{1}{(3-x)(2-x)}$.

6. Solve the equation $x - \frac{x-2}{3} = 5\frac{3}{4} - \frac{x+10}{5} + \frac{x}{4}$.

7. Solve the equation $\frac{2}{x-3} - \frac{24}{4x-3} = \frac{4}{1-x}$.

EXERCISE—XX. (CHAPS. VIII. TO XII.)

1. Simplify $\frac{x+y}{x-y} - \frac{x^2+y^2}{x^2-y^2} - \frac{2x^2y}{x^4-y^4} - \frac{2xy^2}{x^2-y^2}$.

2. Simplify $\frac{a^2-(b-c-d)^2}{(a-b)^2-(c+d)^2}$ and $\frac{a^2+2ab-8b^2+a+4b}{a^2+6ab+8b^2+a+4b}$.

3. Find the L. C. M. of

$$x^4 + x^3 + x^2 + x, \quad x^4 - x^3 + x^2 - x, \quad \text{and} \quad x^4 - x^2.$$

4. Find the H. C. F. of

$$x^5 - 3x^4 + x^3 - 3x^2 + x - 3 \quad \text{and} \quad 7x^4 - 16x^3 - 21x^2 + 9x + 27.$$

5. Solve the equation :

$$\frac{x^3 + ax^2 - bx + c}{x^3 - ax^2 + bx + c} = \frac{x^2 + ax - b}{x^2 - ax + b}.$$

6. Shew that if the number 7 be divided into any two parts, the difference between the squares of the parts is seven times the difference between the parts.

EXERCISE—XXI. (As far as CHAP. XIII.)

1. Find the product of $(p-1)(q-1)(p-q)$; and divide $y^4 - 3y^2 + 9$ by $y^2 + 3y + 3$.

2. If $a + b + c = 0$, prove that $a^2 - bc = b^2 - ca = c^2 - ab$.

3. Reduce to simplest form

$$\frac{31}{12x^2+x-20} - \frac{25}{12x^2+25x+12}.$$

4. Solve the equations :

$$(1) \quad 5(5x-4) - 4(4x-3) + 3(3x-2) - 2(2x-1) = 2.$$

$$(2) \quad 1.2x - .05 = .07x + .3x + 16.55.$$

5. Shew that the cube of the sum of any number and its reciprocal is greater than the sum of their cubes by three times the sum of the numbers.

6. If the numerator and denominator of a fraction be each increased by 3, the fraction becomes 2. If each be increased by 11, it becomes $\frac{3}{2}$. Find the fraction.

EXERCISE—XXII. (As far as CHAP. XIV.)

1. Simplify $a - (b - c) - [a - b - c - 2\{b + c - 3(c - a) - d\}]$.

$$2. \quad \text{Simplify } \frac{\frac{a}{b+c}}{1 + \frac{b}{a+b}} \times \frac{\frac{c}{a+b}}{1 + \frac{c}{b+c}}.$$

3. Find the H. C. F. of

$$x^3 - x^2 + x + 3 \text{ and } x^4 + x^3 - 3x^2 - x + 2.$$

$$4. \quad \text{Solve the equation } .01x + \frac{40 - .5x}{.1} = 1 - x.$$

$$5. \quad \text{If } \frac{a}{b} = \frac{c}{d}, \text{ prove that } \frac{a+nb}{a-nb} = \frac{c+nd}{c-nd}.$$

$$6. \quad \text{Prove that } \frac{x^2+1}{x^2-1} - 1 \div \left(\frac{2x}{2x-1} - \frac{2}{2-x} \right) = \frac{5x}{2(x^2-1)},$$

EXERCISE—XXIII. (As far as CHAP. XIV.)

1. Simplify

$$(-1)^2(b+c) - [(-1)^2(c+a) - (-1)^2\{a+b-3(b-c)\}].$$

2. Multiply

$$x^5 - 2x^4y + 3xy^4 - y^5 \text{ by } y^5 \div 2xy^4 + 3x^4y - x^5.$$

3. Find the H. C. F. of

$$x^3 - x^2(a+4) + x(4a+3) - 3a \text{ and } x^3 - x^2(a-1) - x(a+2) + 2a.$$

4. Find a number such that its half is as much above 100 as its third part is below 100.

5. *A* can do three-fourths of a piece of work in 12 days, *B* can do one-half in 10 days, and *C* can do one-fourth in 4 days. In how many days will they complete their work if all work together?

$$6. \text{ Simplify } \frac{\frac{x^2+xy+y^2}{2} \cdot \frac{2x+3y}{x-y}}{\frac{3}{x-y}} \div \left(\frac{2x+3y}{x^2-y^2} \div \frac{4x^2-9y^2}{x^3-y^3} \right).$$

EXERCISE—XXIV. (As far as CHAP. XVI.)

1. Divide $x^5 - 5x^4 + 9x^3 - 6x^2 - x + 2$ by $x^3 - 2x^2 + x + 1$.

2. Find the L. C. M. of $x+2$, x^2-1 , x^2+x-2 , x^3-3x-4 .

$$3. \text{ Simplify } \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{c}} + \frac{\frac{1}{c} + \frac{1}{a}}{\frac{1}{b}} + \frac{\frac{1}{b} + \frac{1}{c}}{\frac{1}{a}} + 3.$$

4. Solve the equations:

$$(1) \quad -1x + \frac{x-7655}{23} = 7. \qquad (2) \quad \begin{aligned} 5x+7y &= 50 \\ 9x+3y &= 42. \end{aligned}$$

5. If $\frac{bx^2+a^2y}{a} = \frac{ay^2+b^2x}{b}$, prove that $bx+ay=ab$,
or $ay=bx$.

6. One of the digits of a number is greater by 5 than the other. When the digits are inverted, the number becomes $\frac{3}{8}$ of the original number. Find the digits.

EXERCISE—XXV. (As far as CHAP. XVIII.)

1. Find the remainder when $x^7 - 10x^6 + 8x^5 - 7x^4 + 3x - 11$ is divided by $x^3 - 5x + 4$.

2. Find the L.C.M. of

$$6x^3 + x^2y - 11xy^2 - 6y^3 \text{ and } 6x^3 + 11x^2y - xy^2 - 6y^3.$$

3. Find the square root of

$$4x^4 - 12x^3y + 29x^2y^2 - 30xy^3 + 25y^4.$$

4. Find the cube root of $1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6$.

5. Solve the equations :

$$(1) \frac{2x^2 + 4}{5} - \frac{3x^2 - 7}{3} = \frac{11}{15}. \quad (2) \frac{12}{x} + \frac{10}{x-1} = 6\frac{1}{2}.$$

$$(3) \frac{6}{x} + \frac{5}{y} = \frac{44}{xy}; \quad 6x + 5y = 44.$$

$$(4) (x+2)(x+4)(x+6) = x(x+5)(x+7).$$

6. In a division the majority was 162, which was $\frac{3}{11}$ of the whole number of votes : how many voted on each side ?

EXERCISE—XXVI. (As far as CHAP. XIX.)

1. Multiply $\frac{x^3}{y^3} + 1 + \frac{y^3}{x^3}$ by $\frac{x}{y} - \frac{y}{x}$.

2. Solve the equations :

$$(a) x - \frac{x-2}{3} = \frac{x+23}{4} - \frac{10+x}{5}. \quad (b) \frac{3x}{x+1} + \frac{2x-5}{3x-1} = 3\frac{1}{3}.$$

$$(c) \frac{3}{2}(x-1) = \frac{3}{4}(y-1); \quad 2x - \frac{1}{3}y = 7.$$

$$(d) \frac{x+16}{5} + \frac{11}{x} = \frac{4x-17\frac{1}{2}}{3}.$$

3. Find the square root of

$$4a^2x^4 - 12a^2x^3 + 13a^4x^2 - 6a^2x + a^6.$$

4. Find the cube root of

$$x^6 - 9x^5 + 42x^4 - 117x^3 + 210x^2 - 225x + 125.$$

5. Simplify
- $\frac{p^3 - pq + q^3}{\frac{1}{2}ab + cd} \div \left(\frac{ab - 2cd}{p^3 - q^3} \div \frac{a^2b^2 - 4c^2d^2}{p^3 + q^3} \right).$

6. The sum of two digits is 9. Six times one of the numbers they form is equal to five times the other number. Find the digits.

EXERCISE—XXVII. (As far as CHAP. XIX.)

1. If
- $\frac{a}{b} = \frac{c}{d}$
- , shew that
- $\frac{a^3 - 3ab + 4b^3}{c^3 - 3cd + 4d^3} = \frac{a^3 - b^3}{c^3 - d^3}.$

2. Find the fourth root of

$$1 + 4x + 10x^2 + 16x^3 + 19x^4 + 16x^5 + 10x^6 + 4x^7 + x^8.$$

3. Find the cube root of

$$x^6 - 6x^5 + 6x^4 + 16x^3 - 12x^2 - 24x - 8.$$

4. A boat goes up stream 30 miles and down stream 44 miles in 10 hours. Again it goes up stream 40 miles and down stream 55 miles in 13 hours. Find the rates of the stream and the boat.

5. Simplify
- $\frac{ab(x^2 + y^2) + xy(a^2 + b^2)}{ab(x^3 - y^3) + xy(a^3 - b^3)}.$

6. Find the L.C.M. of
- $5x^2 + 19x - 4$
- and
- $10x^2 + 13x - 3.$

EXERCISE—XXVIII. (As far as CHAP. XIX.)

1. Divide
- $x^3 - 3ax^2 + 3a^2x - a^3 + b^3$
- by
- $x - a + b.$

2. Simplify
- $\frac{1}{2} \frac{a^2 + x^2}{a^2 - x^2} - \frac{1}{2} \frac{a + x}{a - x} + \left(\frac{a}{a + x} \right)^2.$

3. Solve the equations :

$$(1) \frac{x-9}{8} - \frac{3}{7}(55-x) + 4 = \frac{x+15}{28}.$$

$$(2) \frac{2 - \frac{1-x}{3}}{5 - \frac{4}{3}} = 1.$$

$$(3) \frac{3}{x+1} + \frac{1}{2(x-3)} = \frac{1}{2}.$$

4. The same amount of tax was paid on the profits of a business when the income-tax was 9*d.* per £1, as when the profits of the business were £100 more, and the income-tax 2*d.* in the £1 less. Find the profits in each case.

5. Six Prussian pounds weigh $\frac{1}{4}$ oz. more than 5 Austrian pounds. Twenty-five Austrian pounds weigh $\frac{1}{4}$ oz. more than 14 kilogrammes. One kilogramme weighs one ounce less than the sum of the weights of a Prussian and an Austrian pound. Find the number of ounces in each foreign measure of weight.

6. Find the H.C.F. of $2a^3 - 2a^2 - 3a - 2$ and $3a^3 - a^2 - 2a - 16$.

EXERCISE—XXIX. (As far as CHAP. XX.)

1. Find the square root of $a^6 + 4a^5 + 2a^4 + 9a^3 - 4a + 4$.

2. Solve the equations :

$$(1) x - \frac{21}{x} + 4 = 0.$$

$$(2) 28x^2 - 19x + 3 = 0.$$

$$(3) \begin{cases} x + 2y = 15 \\ 5x - 19y = 17 \end{cases}.$$

$$(4) \begin{cases} x + \frac{1}{y} = 5.5 \\ y + \frac{1}{x} = 2.2 \end{cases}.$$

3. Income-tax deducted, an estate produced £351. Next year, the rents having increased $\frac{1}{15}$ th, and the income-tax being 1*d.* per £1 less, the estate produced £376. Find the income-tax per £1 in each year.

4. Three thalers are worth $\frac{1}{2}d.$ more than 11 francs. Five francs are worth $\frac{1}{2}d.$ more than 2 florins. One thaler is worth $2d.$ more than a franc and a florin together. Find the value of each coin in English money.

5. If $\frac{a}{b} = \frac{c}{d}$, shew that $\frac{a^2 + b^2}{c^2 + d^2} = \frac{(a-b)^2}{(c-d)^2}$.

6. If $4y^2 - 8x + 1 = 0$, and $x = \sqrt{1 - y^2}$, prove that the values of x are $\frac{1}{2}$ and $-\frac{1}{2}$.

EXERCISE—XXX. (As far as CHAP. XXI.)

1. Simplify $\frac{(a+b+c)^2 + (b-c)^2 + (c-a)^2 + (a-b)^2}{a^2 + b^2 + c^2}$.

2. Solve the equations:

(1) $30x^2 - 49x + 20 = 0$. (2) $x + \frac{1}{2}y = 16$; $\frac{1}{2}x - \frac{1}{3}y = 1$.

(3) $\frac{1}{3}x^2 - \frac{1}{2}x = 5\frac{5}{6}$.

(4) $(x+1)(x+3)(x+5) = (x+7)(x+9)(x-7)$.

3. A number consisted of three digits, the right-hand one being 0. If the left-hand and middle digits be interchanged the number is diminished by 180. If the left-hand digit be halved, and the middle and right-hand digits be interchanged the number is diminished by 454. Find the number.

4. The sides of a rectangle are 12 and 20 feet; what is the breadth of the border which must be added all round that the whole area may be 384 square feet?

5. Find the cube root of

$$m^3 + (3p - 6n)m^2 + (3p^2 - 12pn + 12n^2)m + p^3 - 6p^2n + 12pn^2 - 8n^3$$

6. Resolve into factors $(x^2 + xy - y^2)^2 - (x^2 - xy - y^2)^2$.

EXERCISE—XXXI. (As far as CHAP. XXV.)

1. Multiply $a^{\frac{1}{2}} + a^{\frac{1}{2}}b + a^{\frac{1}{2}}b^2 + a^{-\frac{1}{2}}b^3$ by $a - b$.

2. Divide $a^{\frac{1}{2}} - 2 + a^{-\frac{1}{2}}$ by $a^{\frac{1}{2}} - a^{-\frac{1}{2}}$.

3. Solve the equations:

$$(1) \sqrt{5x+10} - \sqrt{5x} = 2.$$

$$(2) a^{-x}(a^x + b^{-x}) = \frac{a^2b^2 + 1}{a^2b^2}.$$

$$(3) x^2 + y^2 = 90; \quad x + y = 12.$$

4. Find positive integral solutions of $8x - 3y = 16$.

5. The cost price of each copy of a newspaper is $3\frac{1}{4}d.$; the sum for which it is sold is $3d.$, and the number of pence received for the insertion of advertisements is 30 per cent. of the number of copies issued beyond 10,000; find the least number of copies which can be issued without loss.

6. Water expands 10 per cent. when it turns to ice. How much per cent. does ice contract when it turns to water?

7. Reduce to their simplest forms:

$$(1) \frac{6}{a+x} + \frac{7}{a-x} - \frac{x}{a^2-x^2}.$$

$$(2) \frac{\frac{a+b}{a-b} + \frac{a-b}{a+b}}{2(a^2+b^2)}.$$

$$(3) \frac{\frac{x\sqrt{x-1}}{\sqrt{x-1}} + \sqrt{x}}{\sqrt{x+1}}.$$

EXERCISE—XXXII. (As far as CHAP. XXV.)

1. Multiply $x^{\frac{m}{2}} + x^{\frac{m}{2}}y^{\frac{n}{2}} + y^{\frac{n}{2}}$ by $x^{\frac{m}{2}} - x^{\frac{m}{2}}y^{\frac{n}{2}} + y^{\frac{n}{2}}$.

2. Divide $a^{\frac{5}{2}} + a^{\frac{1}{2}}b^{-\frac{1}{2}} - a^{\frac{1}{2}}b^{-\frac{1}{2}} - b^{-\frac{5}{2}}$ by $a^{\frac{1}{2}} - b^{-\frac{1}{2}}$.

3. Solve the equations:

$$(1) \frac{\sqrt{x-3}}{x-6} = \frac{8\sqrt{x-6}}{x-3}.$$

$$(2) 6x^2 = x + 1.$$

$$(3) \sqrt{(a^x)} \sqrt{(a^y)} = a^2; \quad (b^x)^2 = b^2(b^y)^2.$$

4. The registrar of a certain district finds that within a given time a per cent. of the women between 18 and 21 have been married, and $2b$ per cent. of those between 18 and 24; he finds also that the same number have been married between 18 and 21 as between 21 and 24. Shew that the per centage of those between 21 and 24 who have been married is $\frac{ab}{a-b}$.

5. Find the sum and difference of $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ and $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$.

6. Rationalize $\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$.

7. Prove that the sum of any number of even numbers is even, and that the sum of any odd number of odd numbers is odd.

EXERCISE—XXXIII. (As far as CHAP. XXV.)

1. Divide $x^{\frac{3}{2}}+y^{\frac{3}{2}}$ by $x^{\frac{1}{2}}+y^{\frac{1}{2}}$ and $a-b$ by $a^{\frac{1}{2}}-b^{\frac{1}{2}}$.

2. Solve the equations:

$$\left. \begin{aligned} \frac{x+\sqrt{(x^2-y^2)}}{x-\sqrt{(x^2-y^2)}} + \frac{x-\sqrt{(x^2-y^2)}}{x+\sqrt{(x^2-y^2)}} &= \frac{82}{9} \\ \frac{\sqrt{(x^2-y^2)}+y}{\sqrt{(x^2-y^2)}-y} - \frac{\sqrt{(x^2-y^2)}-y}{\sqrt{(x^2-y^2)}+y} &= \frac{48}{7} \end{aligned} \right\}.$$

3. A company of men is formed into a hollow square eight deep, and afterwards into a hollow square four deep; and it is found that the number of men in the front rank of the latter square exceeds the number in the front rank of the former square by 19. Find the number of men in the company.

4. If $2s=a+b+c$, prove that

$$\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s} = \frac{abc}{s(s-a)(s-b)(s-c)}.$$

5. Find positive integral solutions of $27x+16y=1600$.

6. Extract the square root of $20+2\sqrt{91}$.

7. If $x^2 - 2y = -\frac{1}{4}$ and $y = \sqrt{1 - x^2}$, prove that y must be equal to $\frac{1}{2}$ or $-\frac{5}{2}$.

EXERCISE—XXXIV. (As far as CHAP. XXV.)

1. If a number be a factor of two others, prove that it will also be a factor of the difference of any multiples of those numbers.

2. Solve the equations:

$$(1) \quad 7x + \frac{3}{x} = 35\frac{3}{5};$$

$$(2) \quad \left. \begin{aligned} \frac{1}{x - \sqrt{(x^2 - y^2)}} + \frac{1}{x + \sqrt{(x^2 - y^2)}} &= \frac{13}{72} \\ \frac{1}{y - \sqrt{(x^2 - y^2)}} + \frac{1}{y + \sqrt{(x^2 - y^2)}} &= \frac{24}{119} \end{aligned} \right\};$$

$$(3) \quad \sqrt{(2x-1)} + \sqrt{(2x+4)} = 5.$$

3. Two butchers, A and B , buy equal quantities of meat at $6d.$ per lb. A retails it at $8d.$ a pound, and B at $9d.$ After all A 's stock is sold off, he buys what is left of B 's at $7d.$ per pound and sells it at $8d.$ A clears on the whole half as much again as B . What part of his stock did B sell to his customers?

4. If $\frac{x}{a+b-c} = \frac{y}{a-b+c} = \frac{z}{b+c-a}$, shew that each of these fractions $= \frac{x+y+z}{a+b+c}$.

$$5. \quad \text{Simplify } \frac{(yz-a)^2 - (ca-y^2)(ab-z^2)}{(bc-1)(yz-a) - (z-by)(y-cz)}.$$

6. Find the fourth root of $1433 - 432\sqrt{11}$.

7. Divide the number a into two parts so that the product of the parts may if possible be equal to p . Shew that the *greatest* product is when the parts are equal.

EXERCISE—XXXV. (As far as CHAP. XXVI.)

1. Divide $\alpha^3 + \beta^2$ by $\alpha + \beta \sqrt{-1}$.
2. Multiply $m^2 - \sqrt{2} \cdot mn + n^2$ by $m^2 + \sqrt{2} \cdot mn + n^2$.
3. Simplify $\left\{ \frac{x^{m+n}}{x^n} \right\}^m \div \left\{ \frac{x^n}{x^{n+m}} \right\}^{n-m}$.

4. A ladder 34 feet long just reached a window of a house, when placed in such a position that the height of the window above the ground exceeded the distance of the foot of the ladder from the wall by 14 feet. Find the height of the window.

5. Solve the equation $\sqrt{(12x-11)} + \sqrt{(7x+43)} = 13$.
6. Find the square root of

$$x^3 - 2x^{\frac{1}{2}}y^{\frac{1}{2}} + 2x^{\frac{1}{2}}z^{\frac{1}{2}} + y^{\frac{3}{2}} - 2y^{\frac{1}{2}}z^{\frac{1}{2}} + z^{\frac{3}{2}}.$$

7. Solve the equation $x^2 + px + 25 = 0$, and state how its roots differ in character according as p is within or beyond the limits -10 , $+10$.

EXERCISE—XXXVI. (As far as CHAP. XXVIII.)

1. Divide $1 - \frac{a^6}{b^6}$ by $1 - \frac{a}{b}$.

2. A person started at a certain pace to walk to a railway station three miles off, intending to arrive at a certain time; but, after walking a mile, he was detained ten minutes, and was in consequence obliged to walk the rest of the way a mile an hour faster. At what pace did he start?

3. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that $\frac{a^3}{b^3} = \frac{ace}{bdf}$.

4. Solve the equations:

$$(1) \quad x^2 + 7x + 2\sqrt{x^2 + 7x + 8} = 16.$$

$$(2) \quad x^2 - 3xy = 10; \quad xy - 3y^2 = 2.$$

$$(3) \quad x^4 - 45x^2 = 250.$$

5. Resolve into the simplest factors $x^3 + 9x^2 + 11x - 21$.

6. At a contested election there are two members to be returned and three candidates X, Y, Z . X obtains 2112 votes, Y 1974, Z 1866. Now 170 voted for Y and Z , 1500 for Z and X , 316 for X and Y . How many plumped for X, Y, Z respectively?

7. Find the L. C. M. of

$$15x^3 - 44x^2 + 66x - 27 \text{ and } 20x^3 - 67x^2 + 93x - 36.$$

EXERCISE—XXXVII. (As far as CHAP. XXVIII.)

1. Divide

$$4x^{-5} - x^{-4} + 3x^{-3} + 2x^{-2} + x^{-1} + 1 \text{ by } x^{-3} - x^{-1} + 1.$$

2. Divide $x + 3x^{\frac{1}{2}} - 2x^{\frac{1}{4}} - 7x^{\frac{1}{8}} + 2x^{-\frac{1}{4}}$ by $x^{\frac{1}{4}} - 2x^{-\frac{1}{4}}$.

3. Simplify $3\sqrt[3]{128} - 4\sqrt[3]{-686} + 2\sqrt[3]{54}$.

4. The income-tax being $4d$. in the $\pounds$, a person has to pay $\pounds 69$. $5s$. $11d$. less than when the tax was $7d$. in the $\pounds$, although his income has increased by $\pounds 295$. What was his income at first?

5. Two companies of a regiment appeared in the field in strength as $9:11$. At the end of the day the relative strength was as $5:6$. Of the men in the two companies 35 per cent. had been placed hors-de-combat, and 34 of these men belonged to the first company. Find the numbers in each company at first.

6. If $a:b=c:d=e:f$, then $\frac{a^2+e^2}{b^2+f^2} = \frac{a^2+c^2+e^2}{b^2+d^2+f^2}$, and $af+be:cf+de=b:d$.

7. Find the square root of

$$x^{\frac{1}{2}} + 4x^{\frac{1}{4}}y^{\frac{1}{4}} - 2x^{\frac{1}{4}}z^{\frac{1}{4}} + 4y^{\frac{1}{2}} - 4y^{\frac{1}{4}}z^{\frac{1}{4}} + z^{\frac{1}{2}}.$$

EXERCISE—XXXVIII. (As far as CHAP. XXVIII.)

1. Resolve into factors $(m^3 - n^2 + r^2)^2 - 4m^2r^2$.

2. Find the L. C. M. of $x^4 + x^2y^2 + y^4$, $x^3 + y^3$, and $x^3 - y^3$.

3. Form the equation whose roots are 3, -3, and 4.

4. Simplify

$$(1) \frac{a^3 - 39a + 70}{a^2 + 4a - 45}.$$

$$(2) b \frac{a^2 - ab}{a^3 - b^3} \left(a + b + \frac{b^3}{a} \right).$$

$$(3) \frac{x+y}{(y-z)(z-x)} + \frac{y+z}{(z-x)(x-y)} + \frac{z+x}{(x-y)(y-z)}.$$

5. The sum of £17 was divided amongst ten boys and a certain number of girls in such a way that the number of shillings each boy received bore to the number of pence each girl received the ratio of 3 to 5. The share of each girl was 3s. 4d.: find how many girls there were.

6. Shew that if $a:b=c:d=e:f$,

$$27(a+b)(c+d)(e+f) = bdf \left(\frac{a+b}{b} + \frac{c+d}{d} + \frac{e+f}{f} \right)^3.$$

7. Simplify

$$\{ \sqrt{(2x^2 - 4xy + 2y^2)} + \sqrt{(50x^2 - 100xy + 50y^2)} \} \cdot \frac{3}{\sqrt{2} \cdot (x-y)}.$$

EXERCISE—XXXIX. (As far as CHAP. XXVIII.)

1. Resolve into factors $4(ab+cd)^2 - (a^2+b^2-c^2-d^2)^2$.

2. Divide $x^{2n} - y^{2n}$ by $x^{2n-1} + y^{2n-1}$.

3. Find the H.C.F. of $6x^3 - 25x^2 + 34x - 15$ and $9x^2 - 31x + 10$.

4. Simplify:

$$(1) \frac{3x^3 + x^2 - 5x + 21}{6x^3 + 29x^2 + 26x - 21}.$$

$$(2) \frac{p^3 + q^3 - r^3 + 2pq}{r^3 - p^3 - q^3 + 2pq} \div \frac{p+q+r}{q+r-p}.$$

$$(3) \frac{1}{x(x-y)(x-z)} + \frac{1}{y(y-x)(y-z)} + \frac{1}{z(z-x)(z-y)}.$$

5. In a borough the new votes claimed by the two political parties were as 9 : 11. The claims allowed by the revising barrister were as 19 : 25. Of the whole number of claimants 45 per cent. were rejected, and 51 of them belonged to the weaker party. How many new votes were admitted on each side ?

6. Find a fourth proportional to the three quantities $\frac{a}{b}, \frac{b}{c}, \frac{c}{d}$. If $\frac{d}{e}$ be the fourth proportional, then

$$b^4 + a^2c^2 : b^4 - a^2c^2 = d^4 + c^2e^2 : d^4 - c^2e^2 ;$$

and $d=e$, when b is a mean proportional between a and d .

7. A was following B , and after a time B turned, and without changing his pace walked in the opposite direction, and in consequence A approached B five times as fast as before. Compare the rates of A and B .

EXERCISE—XL. (As far as CHAP. XXVIII.)

1. Simplify :

$$(1) \left(\frac{a}{b} - \frac{b}{a} + \frac{b}{c} - \frac{c}{b} + \frac{c}{a} - \frac{a}{c} \right) + (a-b)(b-c)(c-a).$$

$$(2) \frac{2}{x+1} + \frac{2}{x-1} - \frac{4x}{x^2+x+1} - \frac{4x}{x^2-x+1}.$$

2. Find the L. C. M. of

$$x^3 - 6x^2 - 6x - 7 \text{ and } x^3 - 8x^2 + 8x - 7.$$

3. Solve the equations :

$$(i) \left. \begin{aligned} \alpha^x \cdot \alpha^{x+1} &= \alpha^7 \\ \alpha^{2x} \cdot \alpha^{2x+5} &= \alpha^{20} \end{aligned} \right\}.$$

$$(ii) \sqrt{x+a} + \sqrt{x-a} = 2c.$$

4. If $a_1 : b_1 = a_2 : b_2 = \dots = a_n : b_n$, prove that

$$\frac{a_1}{b_1} = \sqrt[n]{\frac{a_1 a_2 \dots a_n}{b_1 b_2 \dots b_n}}.$$

5. I can examine 12 sets of papers which are arranged in order in the same time as 9 sets not so arranged. I examined 500 when 85 per cent. of the men sent up their papers properly arranged. How many could I have examined in the same time if they had all been arranged in order?

6. The Cam has been deepened and widened from Jesus Sluice Lock to the Railway Bridge, a distance of 2 miles, the remaining portion from the Railway Bridge to Baitsbite, a distance of $1\frac{1}{2}$ miles, being at present undredged. An eight-oar which can row a mile in 6 minutes in still water in the undredged part of the river, finds its speed increased 10 per cent. in the dredged portion. Two equally fast eight-oars start at the same time from Jesus Sluice Lock and Baitsbite Lock and meet at the Railway Bridge. Find the rate of the stream.

7. Simplify $\frac{x+y\sqrt{-1}}{x-y\sqrt{-1}} + \frac{x-y\sqrt{-1}}{x+y\sqrt{-1}}$.

EXERCISE—XLI. (As far as CHAP. XXX.)

1. Find a number such that if 5, 11 and 17 be successively subtracted from it the sum of the third, fourth and sixth parts of the respective results shall be equal to 19.

2. Solve the equations :

$$(1) \frac{x-3}{8} - \frac{2}{9}(45-x) + 3 = \frac{x+5}{16}.$$

$$(2) \frac{1 + \frac{1-x}{2}}{\frac{1 - \frac{3}{4}}{4}} = 1.$$

$$(3) \frac{3}{10}x + \frac{5}{6}y = 14.5; \quad \frac{7}{15}x - \frac{3}{8}y = 2.5.$$

3. If y be equal to the sum of two quantities, one of which is constant and the other varies as x^2 , and if when $x=2$, $y=7$, and when $x=3$, $y=12$, find the value of x which will make $y=19$.

4. If a, b, c are in continued proportion, prove that $\left(\frac{b}{c} + \frac{c}{a}\right) : \left(\frac{c}{a} + \frac{a}{b}\right)$ is a ratio of equality.

5. Find the sum of n terms of an arithmetical series whose first term is $\frac{n}{4}$ and whose constant difference is $\frac{n}{2}$.

6. The 5th term of an arithmetic series is 9. Find the sum of 9 terms.

7. A baker charges $9\frac{1}{2}d.$ for a loaf which he represents as weighing 4 lbs., but which really weighs 3 lbs. 12 oz. After he has sold a certain number of loaves, he is detected and fined £5, and thus loses five shillings more than he has cleared by selling short weight. How many loaves does he sell?

EXERCISE—XLII. (As far as CHAP. XXXI.)

1. Solve the equations :

$$(1) \quad \frac{1}{2}(3x+8) - \left\{ x - \frac{1}{7}(x-2) \right\} = x - 3.$$

$$(2) \quad \frac{100-x}{3} = \frac{100+y}{4}; \quad \frac{50-x}{4} = \frac{50+y}{7}.$$

$$(3) \quad 11x^2 + 1 = 4(2-x)^2.$$

2. A person started at the rate of 3 miles an hour to walk to a railway station in order to catch a train, but after he had walked one-third of the distance he was detained 15 minutes, and was obliged in consequence to walk the rest of the way at the rate of 4 miles an hour. How far off was the station?

3. If y be equal to the sum of two quantities, one of which is constant and the other varies inversely as x^2 , and if it be found that when $x=2, y=8$, and when $x=4, y=5$, find the value of y when $x=3$.

4. Sum the following series to n terms :

$$(1) \quad 2+4+6+\dots+2n.$$

$$(2) \quad 1+3+9+\dots+3^{n-1}.$$

$$(3) \quad \text{in which the general term is } 2^n + 3n - 2.$$

5. The product of the first five terms of a geometrical progression is 243. Find the third term.

6. Two vessels contain mixtures of wine and water: in one there is twice as much wine as water, in the other three times as much water as wine. How much must be drawn off from each to fill a pint glass, in order that its contents may be half wine and half water?

7. Find the square root of $41 - 24\sqrt{2}$.

EXERCISE—XLIII. (As far as CHAP. XXXI.)

1. Extract the square root of

$$1 - 4x + 10x^2 - 20x^3 + 25x^4 - 24x^5 + 16x^6.$$

2. What is the coefficient of x^4 in the quotient obtained by dividing $(1 - 2x)^3$ by $(1 + 3x)^2$?

3. Simplify $3\sqrt[3]{54} + 2\sqrt[3]{686} - 4\sqrt[3]{250}$.

4. If x varies as $y + 2z$, and y varies as x , shew that x varies as z .

The per centage of candidates who pass the General Examination varies inversely as the number of days' training for the boat races. When there are 20 days' training, 50 per cent. pass. How many pass out of 250 when there are 25 days' training?

5. The sum of 7 terms of an arithmetical progression is 35. Find the sum of the first and last terms.

6. In a geometrical progression of 7 terms the middle term is the 7th root of the product of all the terms.

7. If $a : b = c : d$ and $a' : b' = c' : d'$, shew that

$$(a) \quad \frac{pa}{b} : \frac{qa'}{b'} = \frac{pc}{d} : \frac{qc'}{d'};$$

(β) if also $ab' = a'b$, then will $cd' = c'd$.

EXERCISE—XLIV. (As far as CHAP. XXXI.)

1. Find to five places of decimals the value of $\frac{4-\sqrt{3}}{4+\sqrt{3}}$.

2. Subtract $\frac{7-2\sqrt{5}}{4-\sqrt{5}}$ from $\frac{15+6\sqrt{5}}{2+\sqrt{5}}$.

3. Solve the equations :

$$(1) \quad \frac{1}{14} \left(3x + \frac{2}{3} \right) - \frac{1}{7} \left(4x - 6\frac{2}{3} \right) = \frac{1}{2} (5x - 6).$$

$$(2) \quad \frac{x^2}{y} + \frac{y^2}{x} = 9; \quad x + y = 6.$$

$$(3) \quad \sqrt{xy} = 18; \quad \sqrt{x} - \sqrt{y} = 3.$$

4. If $p : q = x : y$, prove that

$$(1) \quad 2p + 3q : 2p - 3q = 2x + 3y : 2x - 3y.$$

$$(2) \quad p^2 - q^2 : x^2 - y^2 = pq : xy.$$

5. Prove that if the square of the sum of the first $n-1$ numbers be subtracted from the square of the sum of the first n numbers, the remainder is the cube of n .

6. A wine-merchant has a certain quantity of sherry, worth 30 shillings a gallon, and twice as great a quantity worth 20 shillings a gallon. He mixes them and sells them at 28 shillings a gallon, and the profit is the price of 108 gallons of the mixture. How much has he of each ?

7. Find by summing an infinite geometrical series the time between five and six o'clock, (1) when the hour and minute hands of a common clock are together, (2) when they are opposite.

EXERCISE—XLV. (As far as CHAP. XXXI.)

1. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$, shew that $(x + y + z)^3 = 27xyz$.

2. Simplify

$$\sqrt[3]{x-1} \times \sqrt[3]{x+1} \times \sqrt[3]{x^2+x+1} \times \sqrt[3]{x^2-x+1} \times (x^3-1)^{\frac{1}{3}}.$$

3. The sum of two digits composing a number is 12, and the product of the number by the difference of the digits is 114. Find the number.

4. If the sum of two numbers be multiplied by the sum of their squares the result is 520. If the difference between the numbers be multiplied by the difference between their squares the result is 40. Find the numbers.

5. Shew that the sum of the first seven terms of an arithmetical progression is equal to seven times the middle term.

6. The sum of 8 terms of a geometrical progression : the sum to infinity = 16 : 25. Find the constant factor.

7. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ and p, q, r be any multipliers, prove that each fraction will be equal to $\frac{pa+qc+re}{pb+qd+rf}$ and hence shew

that if $\frac{x}{2a-b+c} = \frac{y}{2b-c+2a} = \frac{z}{2c-a+2b},$

then will $\frac{2x+2y-z}{a} = \frac{2y+2z-x}{b} = \frac{2z+2x-y}{c}.$

EXERCISE—XLVI. (As far as CHAP. XXXI.)

1. How may a bill of £30. 17s. be paid in guineas and sovereigns ?

2. Extract the cube root of

$$27x^3 + 54x^2 + 225x + 260r^3 + 525r^2 + 294r + 343.$$

3. Solve the equations :

$$(1) \quad \sqrt{(12+x)} - \sqrt{x} = 2.$$

$$(2) \quad x^3 + y^3 + z^3 = 29,$$

$$xy + zx + yz = 26.$$

$$x + y - z = 5.$$

4. Find the value of

$$\left(\frac{x-a}{x-b}\right)^3 - \frac{x-2a+b}{x+a-2b}, \text{ when } x = \frac{a+b}{2}.$$

5. Sum $\frac{1}{2}, -\frac{2}{3}, -\frac{11}{6}, \dots$ to 15 terms.

6. Insert 4 geometrical means between 3 and -729 .

7. Find the price of eggs per dozen when two less in a shilling's worth raises the price one penny per dozen.

EXERCISE—XLVII. (As far as CHAP. XXXI.)

1. Sum the following series :

$$(1) \frac{2}{3} + 10\frac{1}{3} + 20 + \dots \text{ to 12 terms.}$$

$$(2) 1 - \frac{5}{8} + \frac{25}{64} - \dots \text{ to infinity.}$$

2. Extract the fourth root of $1400 - 600\sqrt{5}$.

3. Solve the equations :

$$(1) x(x^2 - 7x + 12) = 0.$$

$$(2) \frac{3x + \sqrt{4x - x^2}}{3x - \sqrt{4x - x^2}} = 2.$$

$$(3) 6x + 7y = 5xy,$$

$$3y + 6z = 2yz,$$

$$7z + 27x = 8xz.$$

4. Simplify $\frac{2^n \times (2^n - 1)^n}{2^{n+1} \times 2^{n-1}}$.

5. The sum of £27 was divided amongst three persons so that their shares were in arithmetical progression, and the number of sovereigns in the greatest share was equal to the number of half-crowns in the smallest. Find the shares.

6. The sum of five numbers in arithmetic progression is 15 and the sum of their squares is 55. Find the numbers.

7. In how many ways may a debt of £3. 5s. be paid with crowns and florins?

EXERCISE—XLVIII. (As far as CHAP. XXXI.)

1. Resolve into elementary factors :

(1) $14x^3 - 37x + 5.$

(2) $(1+a)^2(1+c^2) - (1+c)^2(1+a^2).$

(3) $m^4 - n^4 + 2n(m^2 + n^2) - (m+n)^2(m-n)^2.$

2. Multiply

$$a^m - ba^{m-1}x + ca^{m-2}x^2 \text{ by } a^n + ba^{n-1}x - ca^{n-2}x^2.$$

3. Simplify the expressions :

(1) $\left\{ (a^m)^{m-\frac{1}{m}} \right\}^{\frac{1}{m+1}}.$

(2) $\frac{x^{\frac{1}{2}} + 3y^{\frac{1}{2}}}{x^{\frac{1}{2}} - 3y^{\frac{1}{2}}} + \frac{x^{\frac{3}{2}} - 3x^{\frac{1}{2}}y^{\frac{1}{2}} + 9y^{\frac{3}{2}}}{x^{\frac{3}{2}} + 3x^{\frac{1}{2}}y^{\frac{1}{2}} + 9y^{\frac{3}{2}}}.$

(3) $\frac{2}{(x^2-1)^2} - \frac{1}{2x^2-4x+2} - \frac{1}{1-x^2}.$

4. *A* and *B* are two towns situated 18 miles apart on the same bank of a river. A man goes from *A* to *B* in 4 hours, by rowing the first half of the distance and walking the second half. In returning he walks the first half at the same rate as before, but the stream being with him, he rows $1\frac{1}{2}$ miles per hour more than in going, and accomplishes the whole distance in $3\frac{1}{2}$ hours. Find the rates of walking and rowing.

5. Sum $1, -\frac{1}{3}, \frac{1}{9}, \dots$ to infinity, and find the value of the recurring decimal .281313.....

6. In an arithmetic series of which the first term is unity the sum of the 6th and 10th terms is 37. What is the series ?

7. Insert 5 geometrical means between
- $\frac{1}{3}$
- and 243.

EXERCISE—XLIX. (As far as CHAP. XXXI.)

1. Solve the equations:

(1) $\sqrt{9x+1} + \sqrt{4x-3} = 13.$

(2) $30x^2 - 49x + 20 = 0.$

(3) $x^2 + y^2 = 34; x^3 - y^2 + \sqrt{(x^2 - y^2)} = 20.$

2. Insert 5 arithmetic means between 5 and 2.
3. The sum of the odd terms of a geometric series of six terms is 147, and the sum of the even terms is 294. What is the series?
4. A man's marks in an Algebra paper vary directly as the number of lectures he has attended in a month. If he attends twelve lectures he obtains 30 per cent. of full marks. What is the minimum per centage for passing, if he exactly obtains this minimum, having attended ten lectures?
5. Two vessels contain mixtures of wine and water: in one there is three times as much wine as water, in the other five times as much water as wine: how much must be drawn off from each to fill a third vessel which holds 7 gallons, in order that its contents may be half wine and half water?
6. Find a fraction such that if its numerator be diminished by 1, its value becomes $\frac{1}{4}$, and if its numerator and denominator be each increased by 1, its value becomes $\frac{1}{3}$.
7. Find positive integral solutions for the equation

$$7x + 11y = 47.$$

EXERCISE—L. (As far as CHAP. XXXI.)

1. Simplify:

$$(1) (23 - 8\sqrt{7})^{\frac{1}{2}}$$

$$(2) \frac{3\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}}.$$

$$(3) \frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}.$$

2. A wine merchant has two sorts of wine, one sort worth 2 shillings a quart, and the other worth 3s. 4d. a quart; from these he wants to make a mixture of 100 quarts worth 2s. 4d. a quart. How many quarts must he take from each sort?

3. Solve the equations:

$$(1) \frac{21}{1.5x-7} - \frac{10}{x+6} = \frac{4}{x-10},$$

$$(2) 2x + y + 6\sqrt{(2x + y + 4)} = 23,$$

$$4x^2 - 6x = y^2 + 3y.$$

4. Shew that in any arithmetic progression of 5 terms the sum of the first and last terms is in a constant ratio to the sum of the intervening terms.

5. If a varies jointly as b and c , and a is 6 when b is 10 and c is 20, what is a when b is 20 and c is 10?

Six clerks can copy six sheets in six hours: how many can copy a hundred sheets in fifty hours?

6. Resolve into elementary factors $15x^3 - 32x^2 - 25x + 42$.

7. Find the sum of $\frac{9}{8} - \frac{3}{4} + \frac{1}{2} - \dots \text{ad infinitum}.$

PART II.

EXERCISE—LI.

1. If $a=5$, $b=-\frac{1}{2}$, $c=10$, $r=4$, find the value of $\left\{\left(\frac{a}{bc}\right)^r - 1\right\}^a$ and of $\sqrt[3]{(-abc)} - \sqrt[3]{(a^3+c^3)} + \sqrt[3]{(a^3+bc^3+6)}$.
2. Multiply x^3+2x^2-3x-6 by x^3-2x^2-3x+6 , and divide the product by x^4-7x^2+12 . Test the result by making $x=1$.
3. Resolve into their elementary factors :
 (1) $(a^2x+ax^2)^5$. (2) $x^3-2x-195$. (3) $4a^2b^3-(a^2+b^3-c^2)^2$.
4. Find the H.C.F. and the L.C.M. of x^3+4x^2-5
and x^3-7x+6 .
5. The product of two algebraical expressions is

$$\frac{x^3}{y^2} - \frac{2x}{y} - 4 + 4\frac{y}{x} + 4\frac{y^2}{x^2},$$

and one of them is

$$\frac{x}{y} - \frac{2y}{x} :$$

find the other.

- 6. Solve the equations :**

$$(1) \quad \frac{2}{3}(x-5) - \frac{3}{11}\left(x-13\frac{1}{3}\right) = 15 - \frac{3}{5}\left(19 - \frac{x}{3}\right).$$

(2) $19x + 241y = 539$; $133x + 1081y = 2561$.

(3) $x^2 - 18x + 77 = 0$.

7. The sum of two numbers is 10 and the sum of their squares is 58. Find the numbers.

8. What is the number which, if added to each term of the ratio 5 : 3, makes it three-fourths of what it would have become if the same number had been subtracted from each term?

9. Multiply $\sqrt{a} + \sqrt{b} + \sqrt{c}$ by $\sqrt{a} - \sqrt{b} + \sqrt{c}$.

10. Shew that $a + (b + c) = a + b + c$.

EXERCISE—LII.

1. Find the numerical value of

$$\frac{a^3 + b^3}{a^3 - ab + b^3} + \frac{a^3 - b^3}{a^3 + ab + b^3} \text{ when } a = \frac{1}{2} \text{ and } b = -2$$

2. Simplify $\frac{(b-c)a^3 + (c-a)b^3 + (a-b)c^3}{c^3 - bc - ca + ab}$.

3. Solve the equations:

$$(1) \frac{x-1}{x-2} - \frac{x-2}{x-3} = \frac{x-5}{x-6} - \frac{x-6}{x-7}.$$

$$(2) 57x + 23y = 275; 19x - 5y = 3.$$

$$(3) x^2 - \frac{14}{3}x = \frac{32}{9}.$$

$$(4) x^3 + y^3 = 351; x + y = 9.$$

4. Resolve $(ab + cd)(c^2 + d^2) + cd(a^2 + b^2 - c^2 - d^2)$ into two factors.

5. Two numbers are to each other as 2 : 3, but if each is increased by 15, they are to each other as 7 : 9. Find the numbers.

6. Find the square root of

$$9x^4 - 12x^3y + 34x^2y^2 - 20xy^3 + 25y^4$$

7. Sum the series :

(1) 3, 1, -1, ... to 15 terms.

(2) $n+1, 2n+3, 3n+5, \dots$ to n terms.

8. There are two masses, one of gold, the other of copper, such that if a lbs. be added to the former, the resulting masses will be in the ratio $m : n$; but if b lbs. be taken from the latter, the resulting masses will be in the ratio $n : m$. Find the weight of each mass.

9. Find the time between 4 and 5 o'clock when the hour hand is as many minute divisions in advance of the minute hand as the minute hand is past 12.

10. Shew that $a - (b + c) = a - b - c$.

EXERCISE—LIII.

1. Multiply

$$x^3 - 3ax^2 + 4a^2x - 5a^3 \text{ by } a^4 - 2a^3x + 3a^2x^2 - 4ax^3 + 5a^4.$$

2. Divide $a^6 - a^5x + 3a^4x^2 + 3a^3x^4 - ax^5 + x^6$ by $a^2 + ax + x^2$.

3. Simplify

$$(1) \frac{2x^4 - x^3 - 9x^2 + 13x - 5}{7x^3 - 19x^2 + 17x - 5}.$$

$$(2) \frac{x+1}{2(x^2+1)} - \frac{1}{(x^2+1)(x^2+1)} + \frac{1-2x}{3(x^2-x+1)}.$$

4. Solve the equations :

$$(1) \frac{x+3}{4} + \frac{x+1}{2} = 4 + \frac{x-2}{3}.$$

$$(2) \sqrt{x-4} + \sqrt{x} = \frac{2}{\sqrt{x-4}}.$$

$$(3) \frac{2-3x}{4} - \frac{4-x}{x-2} = 2\frac{3}{4}.$$

5. Find the product of $a^{\frac{1}{2}}$, $a^{-\frac{1}{2}}$, $\sqrt[3]{(a^4)}$, $a^{\frac{1}{3}}$, $\sqrt[3]{(a^{\frac{2}{3}})}$ and $(a^{-\frac{1}{3}})^{\frac{1}{2}}$.

6. If $a : b = c : d$, shew that

$$4a^2 - 3ab + 2b^2 : 5a^2 - 2b^2 = 4c^2 - 3cd + 2d^2 : 5c^2 - 2d^2.$$

7. A boy's age was twice the number indicated by the clock, and his father's age was 4 times that of the boy. The sum of their ages was 60. What o'clock was it?

8. The fifth term of an A.P. is 7, and the 9th term is 13, find the 13th term.

9. There is a number consisting of two digits whereof one is the square of the other. The number exceeds that formed by reversing the digits by twice the excess which results in the case of the next whole number when the like operation is performed with it. Find the number.

10. Shew that $a - (b - c) = a - b + c$.

EXERCISE—LIV.

1. If $a=1$, $b=3$, $c=5$, $d=7$, $f=0$, find the numerical value of

$$2ab + \frac{c+d}{a+b} + \frac{2b+2c+f}{(a+b)^2} + 5cdf.$$

2. Shew that

$$x(y+z)(y^2+z^2-x^2) + y(x+z)(x^2+z^2-y^2) + z(x+y)(x^2+y^2-z^2) = 2xyz(x+y+z).$$

3. Reduce

$$\frac{2y + (y^2 - 4)x - 2x^2y}{xy + 2} \text{ to its lowest terms.}$$

4. If

$$x = \frac{a+b-c}{a+b+c},$$

shew that

$$\frac{a+bx^2}{b+ax^2} = \frac{(a-b+c)^2 + 4ab}{(a-b-c)^2 + 4ab}.$$

5. Solve the equations:

$$(1) \frac{1}{2}(x+15) + \frac{2}{19}(24-x) = \frac{3}{7}(2x-3) + \frac{1}{5}(8x+5).$$

$$(2) \frac{300}{x} + 1 = \frac{300}{x-10}.$$

$$(3) \frac{x+2}{7} + \frac{y-x}{4} = 2x-8; \quad \frac{2y-3x}{3} + 2y = 3x+4.$$

6. The difference between the squares of two consecutive numbers is 1503. Find the numbers.

7. Multiply $x^{\frac{1}{2}} + x^{\frac{1}{3}} + 1$ by $x^{\frac{1}{2}} - x^{\frac{1}{3}} + 1$.

8. If $a : b = c : d$, shew that $a(a+b+c+d) = (a+b)(a+c)$.

9. Sum the series $4 + 2\sqrt{2} + 2 + \dots$ to 10 terms.

10. Shew that $xy = yx$.

EXERCISE—LV.

1. Multiply $a+mx-nx^2$ by $a-mx+nx^2$.

2. Find the value of

$$\frac{12x^2 - y^2}{3x^2} + \frac{2x^2}{x+y^2} - \frac{x+y^2+z^2}{5y^2}, \text{ when } x=1, y=3, z=5$$

3. Find the L.C.M. of

$$5(a^2b - ab^2), 35(a^2 + ab^2) \text{ and } 7(ab^2 + b^3).$$

4. Simplify

$$(1) \frac{x-y}{x^2-xy+y^2} + \frac{1}{x+y} - \frac{x^2-xy}{x^2+y^2}.$$

$$(2) \frac{x}{1 - \frac{x}{1 + x + \frac{x}{1-x+x^2}}}.$$

5. Solve the equations :

(1) $x^2 - 3x + 2 = 0$.

(2) $\sqrt{(x+3)} + \sqrt{(x+8)} = 5\sqrt{x}$.

(3) $x^2(x+y) = 80 = x^2(2x-3y)$.

6. If m gold coins are equal in weight to n silver coins, and p of the former equal in value q of the latter, compare the values of equal weights of gold and silver.

7. If $a : b = c : d$,

shew that $a^2 - ab + b^2 : ab + ad - bc = c^2 - cd + d^2 : cd - ad + bc$.

8. Extract the square root of

$$\frac{a^2}{4b^2} + \frac{25b^2}{x^2} - 2\left(\frac{a}{b} + \frac{10b}{a}\right) + 9.$$

9. The first term of an arithmetic series is $n^2 - n + 1$, and the common difference 2, shew that the sum of n terms $= n^2$.

10. Shew that $(x+y)z = xz + yz$.

EXERCISE—LVI.

1. Find the difference between

$$l(m+n)^2 + m(n+l)^2 + n(l+m)^2$$

and

$$(m+n)(l-m)(l-n) + (n+l)(m-l)(m-n) + (l+m)(n-l)(n-m).$$

2. Find the H.C.F. of $x^3 - 6x^2 + 11x - 6$

$$\text{and } x^3 - 9x^2 + 26x - 24.$$

3. Multiply $1 - \frac{2x-3x^2}{4}$ by $1 + \frac{3x-2x^2}{4}$.

4. Solve the equations :

(1) $\sqrt{(x+5)} - \sqrt{(16x+3)} = \sqrt{x}$.

(2) $\frac{4x-17}{x-4} + \frac{10x-13}{2x-3} = \frac{8x-30}{2x-7} + \frac{5x-4}{x-1}$.

(3) $\frac{x-y}{x+y} + \frac{x+y}{x-y} = \frac{10}{3}$; $5y = 1 + 2x$.

5. The sum of the squares of three consecutive *even* numbers is 1460. Find the numbers.

6. Find the cube root of

$$8x^6 - 36x^5 + 66x^4 - 63x^3 + 33x^2 - 9x + 1.$$

7. In a certain A.P. the sum of the first and last terms is one-tenth of the sum of the whole series. Find the number of terms.

8. Multiply $x^{\frac{1}{2}} + x^{\frac{1}{3}} + 1$ by $x^{-3} + x^{\frac{1}{3}} + 1$.

9. If $a : b = b : c = c : d$,

shew that $\sqrt{ab} + \sqrt{bc} + \sqrt{cd} = \sqrt{\{(a+b+c)(b+c+d)\}}$.

10. Give a proof of the Rule of Signs in Multiplication.

EXERCISE—LVII.

1. Prove that $(a^2 + 3a + 1)^2 = a(a+1)(a+2)(a+3) + 1$.

2. Divide $(x^2 - yz)^3 + 8y^3z^3$ by $x^3 - yz$.

3. Find the L.C.M. of

$$a^3 - x^3, a^3 + 3ax + 2x^3 \text{ and } a^3 + ax - 2x^3.$$

4. Simplify the following fractions :

$$(1) \frac{a^2 - (b-c)^2}{(b+c)^2 - a^2} \times \frac{b^2 - (c-a)^2}{(c+a)^2 - b^2} \times \frac{c^2 - (a-b)^2}{(a+b)^2 - c^2}.$$

$$(2) \frac{x}{2x+1} - \frac{x}{1-2x} + \frac{4x^2}{1+4x^2}.$$

5. If $ay + bx = a$ and $by - ax = b$, shew that $x^2 + y^2 = 1$.

6. If $2a + 3b : 3a - 2b = 2c - d : c + 2d$,

shew that $5a + b : a - 5b = 3c + d : 3d - c$.

7. Three numbers are in continued proportion ; the sum of the least and greatest is 51, and the sum of the two greater is 60. What are the numbers ?

8. Divide $x^{\frac{1}{2}}y^{-\frac{3}{2}} + x^{-\frac{1}{2}}$ by $x^{\frac{1}{2}}y^{-\frac{1}{2}} - y^{-\frac{1}{2}} + x^{-\frac{1}{2}}$.

9. A man borrows every year £25, upon which he pays interest at the rate of 4 per cent. per annum; in how long a time will the interest amount to £91?

10. Prove the rule for finding the Highest Common Factor of two algebraical expressions.

EXERCISE—LVIII.

1. If $a=1$, $b=2$, $c=3$, find the numerical value of

$$\frac{a^3 + b^3 - c^3}{ab + ac - bc}.$$

2. Divide $x^4 + 10x^3 + 35x^2 + 50x + 24$ by $x + 4$.

3. Solve the equations:

$$(1) \quad 3x - \frac{x-4}{4} - 4 = \frac{5x+14}{3} - \frac{1}{12}.$$

$$(2) \quad \frac{2x-y}{2} + 14 = 18; \quad \frac{2y+x}{3} + 16 = 19.$$

$$(3) \quad (2x+y)^2 - y = 2(3+x); \quad 2x+3y=5.$$

4. If $x=b+c$, $y=c+a$ and $z=a+b$,

shew that
$$\frac{x^3 + y^3 + z^3 - 3xyz}{a^3 + b^3 + c^3 - 3abc} = 2.$$

5. Two cisterns of equal dimensions are filled with water, and the taps in both are opened at the same time. If the water in one will run out in 5 hours and that in the other in 4 hours, find when one cistern will have twice as much water in it as the other has.

6. Extract the square roots of

$$(1) \quad 49x^8 + 112x^4 + 70x^2 + 64x^3 + 80x + 25.$$

$$(2) \quad x^{\frac{5}{2}} - 4x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 4x - 4x^{\frac{3}{2}} + x^{\frac{5}{2}}.$$

7. Compare the arithmetic and geometric means between $6\frac{1}{4}$ and 16.

8. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f},$

then will $\frac{a^3 + b^3}{c^3e + d^3f} = \frac{b^3(ae + bf)}{d^3(a^3 + b^3)}.$

9. Find a factor which will rationalize $3\sqrt{5} - 5\sqrt{3}.$

10. Prove that $\frac{x}{y} = \frac{mx}{my}.$

EXERCISE—LIX.

1. Shew that $(a^2 + b^2 + 4ab)^2 - (a^2 + b^2)^2 = 8ab(a + b)^2.$

2. Divide $x^3 - 3ax^2 + 3a^2x - a^3 + b^3$ by $x - a + b.$

3. Simplify

$$(1) \frac{x}{x+2y} + \frac{y}{x-2y} + \frac{x-y}{4y^2-x^2}.$$

$$(2) \frac{1}{2(x-1)^2} - \frac{1}{4(x-1)} + \frac{1}{4(x+1)} - \frac{1}{(x-1)^2(x+1)}.$$

4. Find the H.C.F. of

$$6x^4 - 25a^2x^3 - 9a^4 \text{ and } 3x^3 - 15ax^2 + a^2x - 5a^3.$$

5. Shew that if $a : b = c : d, ad + bc : 2bd = a^2 + c^2 : ab + cd.$

6. A can dig a trench in half the time B can, and B can dig it in half the time C can. All working together can dig it in 10 days. How long will each take to dig it by himself?

7. The sixth term of an A.P. is 23, and the eleventh term is 43; find the 18th term, and the first.

8. Simplify $\left(\frac{\sqrt{2} - \sqrt{-2}}{2}\right)^4.$

9. If $A \propto BC$, and when $A=6$, $B=9$, $C=10$; find the value of A when $B=7$, and $C=\frac{2}{21}$.

10. Prove that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

EXERCISE—LX.

1. Divide $x^4 + y^4 - z^4 + 2x^2y^2 - 2z^2 - 1$ by $x^2 + y^2 - z^2 - 1$.

2. Find the H.C.F. of

$$x^5 + x^4 - 8x^3 + 12x^2 - x - 21 \text{ and } x^5 - 3x^3 + 9x^2 - 4x - 3.$$

3. Simplify

$$(1) \left(\frac{a+b}{a-b} + \frac{a-b}{a+b} \right) \div \left(\frac{a+b}{a-b} - \frac{a-b}{a+b} \right).$$

$$(2) \frac{x^6 - y^6}{x^3 + 2xy + y^2} \times \frac{x^3 + y^3}{x^3 - y^2} \times \frac{x+y}{x^4 + x^2y^2 + y^4}.$$

4. If $a : b = c : d$, shew that

$$a^2 + b^2 : ab + ad - bc = c^2 + d^2 : cd - ad + bc.$$

5. What fraction is that to which if $\frac{1}{2}$ be added the result is the same as if it had been multiplied by $2\frac{1}{2}$?

6. Solve the equation :

$$\frac{x+\frac{1}{2}}{x+7} + \frac{\frac{1}{2}-x}{x+5} = \frac{3x+17}{x^2+12x+35}.$$

7. What is the cube root of

$$27x^6 + 54x^5 + 9x^4 - 28x^3 - 3x^2 + 6x - 1?$$

8. Apply the principles of Variation to solve the following question. If 7 men can dig a trench 27 yards long in 36 days working 5 hours a day, how many can dig a trench 81 yards long in 15 days working 9 hours a day?

9. Find the sum of the series $1 - \frac{1}{3} + \frac{1}{9} - \dots$ to n terms.
10. Prove that $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bc}$.

EXERCISE—LXI.

- Simplify $25a - 19b - [3b - \{4a - (5b - 6c)\}] - 8a$.
- Prove that $(a+b)^2 - (b+c)^2 + (c-a)^2 = 3(a+b)(b+c)(a-c)$.
- Divide $x^4 - y^4 + z^4 + 2x^2z^2 - 2y^2 - 1$ by $x^2 - y^2 + z^2 - 1$.
- From $\frac{2x^2+1}{x-1}$ take the sum of $\frac{x+5}{x^2-1}$ and $\frac{2x^2}{x+1}$.
- Solve the equations :
 - $\frac{60-x}{14} - \frac{3x-5}{7} = 6 - \frac{24-3x}{4}$.
 - $\frac{3}{x-1} - \frac{x-4}{x-3} = 1$.
- Find the H.C.F. of $3x^4 - 28x^3 + 58x^2 - 75x + 84$ and $11x^4 - 81x^3 + 34x^2 - 51x + 63$.
- After A has received £10 from B he finds that he has £6 more than B ; they have £40 between them: what sum had each at first?
- If $a : b = c : d = e : f = \dots$
 shew that $a + c + e + \dots : b + d + f + \dots$

$$= (a^n + c^n + e^n + \dots)^{\frac{1}{n}} : (b^n + d^n + f^n + \dots)^{\frac{1}{n}}$$
- A person being asked his age, said that a certain number of years ago ($\frac{1}{n}$ th of his age) he was using continuously a certain figure, which was half as great as the second figure in his age then, and which he had now ceased to use: but after the lapse of the same time as before, he should use that figure again. How old was he?
- Establish the rule for finding the Lowest Common Multiple of two algebraical expressions.

EXERCISE—LXII.

1. Divide

$5x^5 - 108x^3y^2 + 31x^4y + 50x^2y^3 + 16xy^4 + y^5$ by $5x^2 - y^2 - 9xy$.

2. Simplify the expressions :

$$(1) \frac{a+b}{(a-c)(a-b)} + \frac{b+c}{(a-c)(b-c)} + \frac{2b}{(b-c)(b-a)}.$$

$$(2) \frac{x}{x^2+y^2} + \frac{x}{x^2-y^2} + \frac{x^3}{(x-y)(x^2+y^2)} - \frac{2x^3-y^3-xy^3}{x^4-y^4}.$$

3. Solve the equations :

$$(1) 2x-1 + \frac{7x-2}{3} - \frac{3x+4}{5} = \frac{7x-4}{5} + \frac{5x+1}{3}.$$

$$(2) \frac{x}{3} = 5 - 2y; \quad \frac{2x-1}{5} - y + 1 = 0.$$

$$(3) 6x^2 - x = 5.$$

4. Four bells toll at intervals of 3, 7, 12, 14 seconds respectively, and begin to toll at the same instant: when will they next toll together?

5. Find the H.C.F. of

$$x^4 - x^3 - 7x^2 + 8x + 4 \text{ and } x^3 + 5x^2 + 7x + 2.$$

6. A number consisting of two digits is such that when divided by the sum of the digits the result is the second digit, and if the digits be reversed the square of the number thus formed is four times the cube of the sum of the digits. Find the number.

$$7. \text{ If } a : b = c : d = e : f,$$

shew that $ac : ace + bdf + acd = e^2 : e^2 + f^2 + e^2d$.

8. Divide $a^2 - b^2x^2$ by $a + cx$ to four terms.

9. Divide $x - 2(x^{\frac{1}{2}} - x^{-\frac{1}{2}}) + 2(x^{\frac{5}{2}} - x^{-\frac{5}{2}}) - x^{-1}$ by $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$.

10. Shew that $\frac{m}{n} + \frac{p}{r} = \frac{mr + np}{nr}$.

EXERCISE—LXIII.

1. Simplify

$$(a+b+c)(a+b+d) + (a+c+d)(b+c+d) - (a+b+c+d)^2.$$

2. Find the L.C.M. of

$$21x^2 - 13x + 2, \quad 28x^2 - 15x + 2 \quad \text{and} \quad 12x^2 - 7x + 1.$$

3. Shew that

$$\frac{p+q+r}{p-q} - \frac{q(4p+3r)-r(p+r)}{p^2-q^2} = \frac{(p-q+r)^2}{p^2-q^2}.$$

4. Solve the equations :

$$(1) \quad \frac{4x-5}{18} - \frac{6x-13}{10} = \frac{4}{15}.$$

$$(2) \quad 5x + 6x - 8 = 75x + 25.$$

$$(3) \quad x^2 - \frac{11}{12}x - 3 = 0.$$

5. Divide $a^{\frac{4}{3}} - 2 + a^{-\frac{4}{3}}$ by $a^{\frac{2}{3}} - a^{-\frac{2}{3}}$.

6. Says *A* to *B*: "Thirteen years ago my age was four times yours, but in thirteen years' time it will be but double your brother's, who was not then born and who is eight years your junior." How old was *A* when *B*'s brother was born?

7. The difference between two numbers is 8, and their product 240. What are the numbers?

8. A suspension bridge is constructed over certain straits which are $2\frac{53}{68}$ miles wide. Each chain of the bridge is to be longer by 12 links than the width of the straits. The length of a link in feet : the number of links in each chain = 5 : 8. Find the number of links in each chain.

9. Find the square root of $82 + 10\sqrt{21}$.10. Shew that $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.

EXERCISE—LXIV.

1. Divide $x^3 - x^2 - 2$ by $x^2 - x + 1$.
2. Find the H.C.F. and L.C.M. of $x^2 + 2x - 120$ and $x^3 - 2x - 80$.

3. Shew that

$$(1) \quad (x-y)^2 - x^2 + y^2 = 3xy(y-x).$$

$$(2) \quad a^2 - 2b^2 = 2(a-b)^2 - (a-2b)^2.$$

$$(3) \quad \left(1-x \cdot \frac{4-x}{4}\right) \left(1+x \cdot \frac{4+x}{4}\right) = \left(1-\frac{x^2}{4}\right)^2.$$

4. Simplify

$$(1) \quad \frac{x-a}{x-b} - \frac{x-b}{a-x} - \frac{(a-b)^2}{(x-a)(x-b)}.$$

$$(2) \quad 1 - \frac{x^2 - (y+z)^2}{(x+y+z)^2}.$$

5. Solve the equations :

$$(1) \quad \frac{x}{3} + 2x \left(\frac{3}{2} - \frac{1}{6}\right) - 1 = 2\frac{1}{2}.$$

$$(2) \quad 10x = 11 - \frac{y+5}{7}; \quad 8y = 17 - \frac{x+3}{4}.$$

$$(3) \quad \frac{3x+2}{x-3} - \frac{3x-2}{x+3} = \frac{4x+36}{x^2-9}.$$

6. What number exceeds its square root by 306 ?

7. If $a : b = c : d = e : f$, shew that

$$a - mc + e : b - md + f = ma - nc + qe : mb - nd + qf.$$

8. Find the continued product of

$$\sqrt{a} + \sqrt{b} + \sqrt{c}, \sqrt{a} + \sqrt{b} - \sqrt{c}, \sqrt{a} - \sqrt{b} + \sqrt{c}, \sqrt{b} + \sqrt{c} - \sqrt{a}.$$

9. A man receives wages at so much a month, which are to increase by one shilling every month till they reach £3. At the end of the first month when he receives £3 they have averaged 12 shillings a week. How long has he been receiving them ?

10. Establish the rule for finding the Square Root of an algebraical expression.

EXERCISE—LXV.

1. The product of two algebraical expressions is

$$4x^5 + 11x^4 + 81, \text{ and one of them is } 4x^3 + 3x^2 - 18x + 27.$$

Find the other.

2. Divide $a^4(b-c) + b^4(c-a) + c^4(a-b)$ by

$$(a-b)(b-c)(c-a).$$

3. If $x+y+z=0$, shew that $x^3+y^3+z^3=3xyz$.

4. Solve the equations:

$$(1) \sqrt{x+18} + \sqrt{x-18} = 6.$$

$$(2) \frac{2}{x+12} + \frac{5}{x+9} = \frac{3}{x+15} + \frac{4}{x+6}.$$

$$(3) (x-9)(x-7)(x-5)(x-1) = (x-2)(x-4)(x-5)(x-10).$$

5. From the expression $(a+b)(a-b) = a^2 - b^2$, find the products of 79 by 81 and $118\frac{1}{2}$ by $121\frac{1}{2}$.

6. I want to fill a shelf two feet long with books, some of which are two inches and others 3 inches thick. In how many ways can I do it?

7. Prove that if the cube of any number less than 100 be added to the cube of the number formed by reversing its digits, the sum will be divisible by eleven times the sum of the digits of the original number.

8. If $a : b = c : d$, shew that

$$ma + nc : mb + nd = (a^2 + c^2)^{\frac{1}{2}} : (b^2 + d^2)^{\frac{1}{2}}.$$

9. Extract the square root of

$$x^{\frac{1}{2}} - 2a^{-\frac{1}{2}} x^{\frac{1}{2}} + 2a^{\frac{1}{2}} x^{\frac{1}{2}} + a^{-\frac{1}{2}} x^{\frac{1}{2}} - 2a^{\frac{1}{2}} x^{\frac{1}{2}} + a^{\frac{1}{2}}.$$

10. Establish the rule for finding the Cube Root of an algebraical expression.

EXERCISE—LXVI.

1. Find the H.C.F. of $a^4 + 2a^3b - 7a^2b^2 + 8ab^3 - 5b^4$ and $a^5 + 4a^4b - 4a^3b^2 - 14a^2b^3 + ab^4 + 15b^5$.

2. Solve the equations:

$$(1) \frac{x-4}{x-5} - \frac{x-5}{x-6} = \frac{x-7}{x-8} - \frac{x-8}{x-9}.$$

$$(2) ax + by = mxy; \quad \frac{a}{x} + \frac{b}{y} = n.$$

3. A and B can do a piece of work in 8 days, B and C can do it in 12 days, and A , B , and C can do it in 6 days. In how many days can A and C do it?

4. Simplify

$$(1) \frac{x^3 - 11x^2 + 39x - 45}{3x^2 - 22x + 39}.$$

$$(2) \frac{x - 4 - 3x^{\frac{1}{2}} + 4y^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}}}{x - 8 - 2x^{\frac{1}{2}} + 12y^{\frac{1}{2}} - 3x^{\frac{1}{2}}y^{\frac{1}{2}}}.$$

5. If $a^2 + b^2 = 1 = c^2 + d^2$, shew that $(ac - bd)^2 + (ad + bc)^2 = 1$.

6. If the constant difference of an A.P. is equal to twice the first term, prove that the quotient of the sum of the series divided by the first term is a perfect square.

7. If $a, b, c, d, \dots, l$ be n terms in a Geometrical Progression, shew that $(abcd \dots l)^{\frac{1}{n}} = (al)^{\frac{1}{n}}$.

8. Multiply $x + y^{\frac{1}{2}} + z^{\frac{1}{2}}$ by $x - y^{\frac{1}{2}} + z^{\frac{1}{2}}$.

9. Find the square root of $41 + 24\sqrt{2}$.

10. What is meant by the symbol a^m ? What is meant by the symbol ma ?

EXERCISE—LXVII.

1. Divide $x^6 + px^5 + (p - q + r)x^4 + (p^2 - pq)x^3 + (pq + pr - qr)x^2 + p^2qx + pqr$ by $x^3 + px + r$.

2. Find the H. C. F. of

$$x^5 + 6x^3 - x^2 + 12x - 36 \text{ and } x^3 + x^2 + x - 6.$$

3. Simplify

$$(1) \frac{x^3 - x + 6}{x^4 - 5x^3 + 4x^2 + x - 15}.$$

$$(2) \left(\frac{a}{b} + \frac{b}{a}\right)\left(\frac{c}{d} + \frac{d}{c}\right) - \left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{c}{d} - \frac{d}{c}\right).$$

$$(3) \left\{\frac{p^2x^3}{qs} + \frac{pqx^2}{r^2s} - \frac{prx}{s^2} - \frac{q^2x}{rs^2} + \frac{p^2x}{qr} - \frac{p}{s}\right\} \div \left\{\frac{px}{r} - \frac{q}{s}\right\}.$$

4. Divide $x + 2x^{\frac{5}{3}} - 7x^{\frac{1}{3}} - 12x^{\frac{2}{3}} + 5$ by $x^{\frac{1}{3}} - 3x^{\frac{2}{3}} + 1$.

5. The difference between two numbers is 8, and the sum of their squares is 544. Find the numbers.

6. A sum of money consists of shillings, half-crowns and sixpences; it is worth as many shillings as there are pieces of money; it is also worth as many half-crowns as there are shillings, and the number of sixpences is one more than the number of shillings. Find the number of each coin.

7. Solve the equations:

$$(1) (x-5)(x^2-5x+5) + x = 5.$$

$$(2) x^3 + 3xy = 55; xy - y^3 = 6.$$

8. *A* challenges *B* to an archery match, agreeing that he shall lose unless he doubles *B*'s score. During the match the scores being counted, it appears that *B*'s is ten more than the half of *A*'s; but after this *B* only scores two for three that he did before; and consequently by the time that *A*'s score has reached double what it was, *A* has won the match, having

exactly doubled B 's whole score. Required the scores when they were first counted.

9. If α and β be the roots of the equation $x^2 + px + q = 0$, shew that $\alpha^2 + \beta^2 = 3pq - p^2$.

10. Shew that $a^m \times a^n = a^{m+n}$, when m and n are positive integers.

EXERCISE—LXVIII.

1. Remove the brackets from the following expressions :

$$(1) \quad ad - [dc + d\{a - (c + b) - (d - \{c + b\} - c)\}].$$

$$(2) \quad x^4 - [4x^3 - \{6x^2 - (4x - 1)\}] - (x^4 + 4x^3 + 6x^2 + 4x + 1).$$

2. Divide

$$12a^4 - 26a^3b - 8a^2b^2 + 10ab^3 - 8b^4 \text{ by } 3a^2 - 2ab + b^2.$$

3. Find the H. C. F. of

$$3x^3 - 13x^2 + 23x - 21 \text{ and } 6x^3 + x^2 - 44x + 21.$$

4. Simplify the fractions :

$$(1) \quad \left\{ \frac{3}{x-1} + \frac{2}{x-2} - \frac{2}{x-3} \right\} + \left\{ \frac{6}{x-1} + \frac{6}{x-2} - \frac{5}{x-3} \right\}.$$

$$(2) \quad \frac{6x^2 - 7x - 20}{4x^3 - 27x + 5}.$$

5. If $a : b = c : d$, shew that $a : b = (\sqrt{a} + \sqrt{c})^2 : (\sqrt{b} + \sqrt{d})^2$.

6. A and B set out to meet each other from places which are distant from each other 1425 miles. A travels 5 miles the first day, 7 the second, 9 the third, and so on; B travels 4 miles the first day, 6 the second, 8 the third, and so on: in how many days will they meet?

7. Find the cube root of

$$x^6 + 3ax^3 + 3bx + 3a^3 + \frac{6ab}{x^2} + \frac{a^3}{x^3} + \frac{3b^2}{x^4} + \frac{3a^2b}{x^5} + \frac{3ab^2}{x^7} + \frac{b^3}{x^9}.$$

8. If α and β be the roots of the equation $px^2 + qx + r = 0$, shew that

$$\alpha + \beta = -\frac{q}{p} \quad \text{and} \quad \alpha\beta = \frac{r}{p}.$$

9. If $t = \frac{1}{1-\sqrt{x}}$ and $s-t = s \cdot \frac{t-1}{t}$, shew that

$$s = 1 + 2\sqrt{x} + 3x + 4x^{\frac{3}{2}} + \dots$$

10. Shew that $(a^m)^n = a^{mn}$ when m and n are positive integers.

EXERCISE—LXIX.

1. Find the value of

$$\frac{x^3 + ax + a^3}{x^3 - a^3} - \frac{x^3 - ax + a^3}{x^3 + a^3} \quad \text{when} \quad x = -\frac{4a}{5}.$$

2. Find the H. C. F. of

$$x^4 - px^3 - p^2x^2 - p^3x - 2p^4 \quad \text{and} \quad 3x^3 - 7px^2 + 3p^2x - 2p^3.$$

3. Simplify

$$(1) \frac{a^2 + a + b^2 + b + c^2 + c + 2ab + 2ac + 2bc}{a^2 - b^2 - c^2 - 2bc}.$$

$$(2) \left\{ \frac{a^6b^3 - 2a^4b^4 + a^2b^6}{(a-b)^6 + 8ab(a-b)^4 + 16a^2b^2(a-b)^2} \right\}^{\frac{1}{2}}.$$

4. Solve the equations:

$$(1) \frac{x+b}{\sqrt{x}} = \frac{a+b}{\sqrt{a}}. \quad (2) \frac{\sqrt{12-x}}{5} = \frac{3}{2+\sqrt{12-x}}.$$

$$(3) \frac{x^3-1}{(x-1)^3} - \frac{x^3+1}{(x+1)^3} = 6. \quad (4) \left. \begin{aligned} x^3 + xy &= 36 \\ y^3 - xy &= 5 \end{aligned} \right\}.$$

5. A mixture of a certain quantity of brandy with 20 gallons of water is worth 25 shillings per gallon. If the brandy be worth 30 shillings per gallon, how much brandy is there in the mixture?

6. Extract the square root of

$$x^4 + \frac{5}{2}x^2y^2 + y^4 - xy(x^2 + y^2)\sqrt{2}.$$

7. If $a : b = c : d$, shew that

$$\frac{a^3 - 3a^2b - 3ab^2 + b^3}{c^3 - 3c^2d - 3cd^2 + d^3} = \frac{a^3 + b^3}{c^3 + d^3}.$$

8. Sum the series $-9 - 7 - 5 - \dots$ to 20 terms.

9. Sum the series $\frac{2}{3} + \frac{1}{6} + \frac{1}{24} + \dots$ to infinity.

10. Discuss the meaning of the symbol $a^{\frac{p}{q}}$.

EXERCISE—LXX.

1. Express in factors

$$(1) 1 - \frac{2bc + c^2}{a^2 - b^2} \text{ and } (2) 1 - \frac{a^2 + b^2 - c^2}{2ab}.$$

2. Simplify $\frac{x^{2n}}{x^n - 1} - \frac{x^{2n}}{x^n + 1} + \frac{1}{x^n + 1} - \frac{1}{x^n - 1}.$

3. Solve the equations :

$$(1) \frac{x+1}{7} + (x-2)x = (x-1)^2.$$

$$(2) (x+11)^{\frac{1}{2}} + (x+11)^{\frac{1}{4}} = 6.$$

$$(3) x^2 + 5xy = 14; y^2 + 6xy = 13.$$

4. I have a sum of money, consisting of sovereigns and shillings, in my purse, which is worth 13 crown pieces: but if the sovereigns were shillings and the shillings were sovereigns, I should want 4 shillings of being better off by 21 guineas. What is the state of my purse?

5. If $x = \frac{1}{1-y}$ and $y = \frac{z}{1-z}$, shew that

$$z = 1 + x + 2x^2 + 4x^3 + \dots$$

6. If $\frac{a+b}{a-b} = \frac{c}{d}$, shew that $\frac{a^2+ab}{ab-b^2} = \frac{c^2+cd}{cd-d^2}$.
7. If a^2, b^2, c^2 be in A.P. shew that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.
8. Apply the principles of Variation to solve the following question. Nine horses having 4 feeds a day can be kept for 3 weeks for £12. 8s. 0 $\frac{1}{2}$ d.: what will be the cost of 15 horses for 36 days with 3 feeds a day?
9. Find the square root of $25 - 4\sqrt{21}$.
10. Shew that $a^0 = 1$.

EXERCISE—LXXI.

1. Find the H.C.F. of $x^4 + 4x^2 + 16$ and $x^4 - x^3 + 8x - 8$.
2. Resolve $4(xy+ab)^2 - (x^2+y^2-a^2-b^2)^2$ into four factors.
3. Solve the equations:
 (1) $axy = c(bx+ay); bxy = c(ax+by)$.
 (2) $x^2(ax+3by) = 4ab^2; y^2(3ax+by) = 4a^2b$.
4. A starts from C and travels towards D at a rate of 6 miles per hour: two hours afterwards B starts also from C , and going 10 miles per hour reaches D four hours before A . Find the distance between C and D .
5. If $x : y : z : \&c. = a : b : c : \&c.$, prove that

$$\frac{xy+yz+zx+\&c.}{ab+bc+ca+\&c.} = \frac{x^2+2y^2+3z^2+\&c.}{a^2+2b^2+3c^2+\&c.}$$
6. Shew that $a^3+b^3 : a^2+b^2$ is greater than $a^2+b^2 : a+b$.
7. Sum the series $\frac{3}{4} + \frac{1}{16} + \frac{1}{192} + \dots$ to 11 terms.
8. Simplify $\frac{a^x+b^y}{a^{-x}+b^{-y}} \times \frac{a^y-b^x}{a^{-y}-b^{-x}}$ and $\frac{e^{2x}-1}{1-e^{-2x}}$.

9. Form the equation whose roots are 0, 1, 2, -2.

10. Shew that $a^{-n} = \frac{1}{a^n}$.

EXERCISE—LXXII.

1. Find the L. C. M. of $x^3 - a^2 - ax(x-a)$, $x^3 - a^2$, and $(x+a)^2$.

2. Find the square root of

$$9x^4 + 6(a+b)x^3 + (a^2 + 2ab + 7b^2)x^2 + 2(ab^2 + b^3)x + b^4.$$

3. Solve the equations:

$$(1) \frac{x-7}{14} - \frac{2x+4}{7(x-3)} = \frac{\frac{3}{2}x-9}{21}.$$

$$(2) (x-1)(x-2) + (x-2)(x-3) + (x-3)(x-4) = 13\frac{1}{4}.$$

$$(3) \frac{x}{2b} + \frac{y}{a} = \frac{1}{6}; \quad \frac{x}{8a} + \frac{y}{24b} = \frac{1}{2}.$$

4. A trader bought two pieces of calico of different quality for 30 shillings, a yard of the finer sort costing him as many shillings as there were yards in that piece. There were 28 yards of the coarser sort, and 8 yards of this cost as much as one yard of the finer. Required the value of each piece.

5. If $z^2 = x^2 + y^2$, reduce

$$\frac{xy}{\sqrt{(x+y+z)(x+y-z)(x+z-y)(y+z-x)}}$$

to its simplest form.

6. If $a : b = c : d = e : f$, prove that

$$ma^2 + nce : mb^2 + ndf = (c+e)^2 : (d+f)^2.$$

7. Simplify $\frac{20+30\sqrt{2}}{3+\sqrt{3}} - \frac{5-2\sqrt{2}}{2-\sqrt{3}}$.

8. Find the cube of $(a+x)^{\frac{1}{2}} - (a-x)^{\frac{1}{2}}$.

9. If a man saves £300 in one year, £400 in the next, £500 in the next, and so on, till he has saved £5200, how long has he been saving?

10. Shew that $a^m \div a^n = a^{m-n}$.

EXERCISE—LXXIII.

1. Reduce

$$(x+y+z)^2 + (x-y-z)^2 + (y-z-x)^2 + (z-y-x)^2.$$

2. Find by involution the square of $x+a^{-1}-1$ and the cube of $a-2a^{-1}+3$.

3. Find the value of

$$\frac{\sqrt{(x^2+a^2)} + \sqrt{(x^2-a^2)}}{\sqrt{(x^2+a^2)} - \sqrt{(x^2-a^2)}} \text{ when } x = \sqrt{\left(\frac{a^4+1}{2}\right)}.$$

4. Solve the equations $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 4$; $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 28$.

5. If $a:b=c:d$, shew that

$$a^2+ab:c^2+cd=ab-b^2:cd-d^2.$$

6. Two casks *A* and *B* contain mixtures of wine and water; in *A* the quantity of wine is to the water as 4:3; in *B* the like ratio is 2:3. If *A* contains 70 gallons, what must *B* contain, so that when the two are put together the new mixture may be half wine and half water?

7 Simplify:

$$(1) \left(\frac{x^2}{y^2}-1\right)\left(\frac{x}{x-y}-1\right) + \left(\frac{x^2}{y^2}-1\right)\left(\frac{x^2+xy}{x^2+xy+y^2}-1\right).$$

$$(2) 4\sqrt{(48)}-2\sqrt{(98)}+11\sqrt{(243)}-10\sqrt{(300)}+7\sqrt{8}.$$

$$(3) \sqrt{\left\{\frac{22+6\sqrt{5}}{8-3\sqrt{5}}\right\}}.$$

8. *A* and *B* walk over the same ground, going out one way and coming home the other, but they start in different directions. *A* walks $3\frac{3}{4}$ miles per hour, and *B* walks 4 miles

per hour. *A* wants $\frac{1}{4}$ of a mile of being half-way when he meets *B*. Required the length of the walk.

9. Shew that if $2s = a + b + c$,

$$\frac{(s-b)(s-c)}{bc} + \frac{(s-a)(s-c)}{ac} + \frac{(s-b)(s-c)}{ba} + 2 \cdot \frac{(s-a)(s-b)(s-c)}{abc} = 1.$$

10. Prove that $(ab)^{\frac{1}{a}} = a^{\frac{1}{a}} b^{\frac{1}{a}}$.

EXERCISE—LXXIV.

1. Resolve into factors:

(1) $x^2 + x - 182$.

(2) $(a^2 - 2b^2 - c^2)^2 - 4(b^2 - c^2)^2$.

(3) $2x^2 + 3x - 2$.

2. Solve the equations:

(1) $(x+3)(x+2)(x+5) = (x-3)(16x+5) + (x+3)(x-4)(x-5)$.

(2) $\left(x - \frac{1}{2}\right)\left(x - \frac{1}{4}\right) - \left(x + \frac{2}{3}\right)\left(x - \frac{2}{5}\right) + \frac{197}{120} = 0$.

(3) $\frac{x-1}{x-2} + \frac{1}{x} - \frac{1}{4} = 0$.

3. Extract the cube root of

$$8x^6 - 36x^5y + 102x^4y^2 - 171x^3y^3 + 204x^2y^4 - 144xy^5 + 64y^6.$$

4. A bill of £24 is paid in half-crowns and half-sovereigns, so that there are 99 coins in all. How many are there of each?

5. Shew that if $a : b = c : d$,

(1) $m(a+b) + nb : m(c+d) + nd = a : c$.

(2) $a^4 + c^4 : b^4 + d^4 = a^2c^2 : b^2d^2$.

6. A man setting out to climb a mountain rises in consecutive intervals of 2 hours each through heights of 2000, 1600, 1200..... feet respectively. How soon will he get to the height of 5600 feet? Explain the double answer.

7. The sum of three numbers in Geometrical Progression is $\frac{13}{15}$ and the sum of their reciprocals is $\frac{65}{3}$. Find the numbers.

8. Simplify $\frac{a+b\sqrt{-1}}{a-b\sqrt{-1}} + \frac{a-b\sqrt{-1}}{a+b\sqrt{-1}}$.

9. Write down the remainder after the p^{th} term of the quotient in the division of $x^m + a^m$ by $x + a$.

10. Shew that if m is a whole or fractional number, and $\sqrt{n}$, $\sqrt{r}$ surds, $\sqrt{r}$ cannot be equal to $m + \sqrt{n}$, unless $m = 0$.

EXERCISE—LXXV.

1. Divide $a^4 - 29a^3 - 50a - 21$ by $a^2 - 5a - 7$.

2. Find the H. C. F. of

$$x^5 - 5ax^4 + 2a^2x^3 + 32a^3x^2 - 70a^4x + 49a^5 \text{ and } \\ x^4 - 8ax^3 + 33a^2x^2 - 76a^3x + 77a^4.$$

3. Simplify

$$(1) \frac{3a(x^3 + ax + a^2)}{x^3 - a^3} - \frac{2x^3 + 3ax^2 - 5a^3}{x^3 + a^3} - \frac{3a}{x - a} + \frac{3ax - 3a^2}{x^2 - ax + a^2}$$

$$(2) \frac{(a^{\frac{1}{2}})^{\frac{1}{2}}(b^{\frac{1}{2}})^{\frac{1}{2}}}{(b^{\frac{1}{2}})^{\frac{1}{2}}a^{\frac{1}{2}}} \times \frac{(a^{\frac{1}{2}})^{\frac{1}{2}}(b^{\frac{1}{2}})^{\frac{1}{2}}}{(b^{\frac{1}{2}})^{\frac{1}{2}}}$$

4. Find the cube root of

$$x^9 - 21x^7 + 4x^6 + 147x^5 - 56x^4 - \frac{1013}{3}x^3 + 196x^2 - \frac{112}{3}x + \frac{64}{27}.$$

5. Simplify
$$\frac{(xy)^{a+b} + x^by^a - x^ay^b - 1}{\left(\frac{y}{x}\right)^a (xy)^b + y^{a+b} \{(xy)^b - (xy)^{-a}\} - y^{2b}}$$

6. Solve the equations :

$$(1) \quad xy = ab ; (x+y)(a-b) = (x-y)(a+b).$$

$$(2) \quad \left. \begin{aligned} 2.4x + .32y - \frac{.36x - .05}{.5} &= 8x + \frac{2.6 + .005y}{.25} \\ \frac{.04y + .1}{.3} &= \frac{.07x - .1}{.6} \end{aligned} \right\}.$$

$$(3) \quad (x+2y)(2x+y) = 20 ; 4x(x+y) = 9 - y^2.$$

7. *A* and *B* set out at the same time from two places 247 miles distant from each other. *A* travels at the rate of 9 miles an hour, and *B* at such a rate that the number of miles he travels per hour is 3 less than the number of hours at the end of which they meet. When will they meet?

8. Find the sum of the series $1 + 3 + 9 + \dots$ to 12 terms.

9. If $\frac{5x}{7} + 4$ be not greater than 8, and $\frac{4x}{3} + 5$ not less than 10, find the integral value of x .

10. If a and b are whole or fractional numbers and $\sqrt{c}$, $\sqrt{d}$ surds, and if also $a + \sqrt{c} = b + \sqrt{d}$, shew that $a = b$ and $c = d$.

EXERCISE—LXXVI.

1. Find the H.C.F. of $6(a^3 - b^3)(a - b)^2$, $9(a^4 - b^4)(a - b)^2$ and $12(a^3 - b^3)^2$.

2. Find the square root of $49x^4 + 56x^2y + 30x^2y^2 + 8xy^3 + y^4$.

3. Solve the equations :

$$(1) \quad x^2 - 34 = \frac{x}{3}.$$

$$(2) \quad \sqrt{(2x+1)} + \sqrt{(7x-27)} = \sqrt{(3x+4)}.$$

$$(3) \quad \left(\frac{x}{a} - \frac{2a}{x} - 1 \right) \left(1 + \frac{a}{x} - \frac{2x}{a} \right) = 0.$$

$$(4) \quad \frac{x}{y} + \frac{y}{x} = \frac{5}{2}; \quad xy = 8.$$

4. A cannon-ball leaves a gun at such a velocity in feet per second that its half, its fourth and its tenth parts together equal the difference between seven times its eighth part and 39. Find its velocity.

5. Sum the following series :

$$(1) \frac{27}{16} + \frac{18}{15} + \frac{57}{80} + \dots \text{ to 6 terms.}$$

$$(2) 3\frac{3}{8} + 2\frac{1}{4} + 1\frac{1}{2} + \dots \text{ to infinity.}$$

6. Simplify

$$(1) \frac{x^3 + 2ab - a^2 - b^2}{x^2 - ax + ab - b^2}, \text{ and } (2) \frac{t^{\frac{2\pi}{3}} - t^{-\frac{2\pi}{3}}}{t^{\frac{\pi}{3}} - t^{-\frac{\pi}{3}}}.$$

7. If $a : b = c : d = e : f$, shew that

$$ac + ce + ea : bd + df + fb = a^2 + c^2 + e^2 : b^2 + d^2 + f^2.$$

8. If $a + 3 : b + 2 = 9(b - 2) : 4(a - 3)$, find the ratio $a : b$.

9. Form the equation whose roots are 2 , $\frac{1}{2}$ and $-\frac{1}{2}$.

10. Shew how to find the square root of $a + \sqrt{b}$.

EXERCISE—LXXVII.

1. Find the H. C. F. of

$$a(cd - be)^2 + 2b(cd - be)(ae - bd) + c(ae - bd)^2$$

and $a(cd + be)^2 - 2b(cd + be)(ae + bd) + c(ae + bd)^2.$

2. Multiply together $ax - c$, $bx - c$, $c^2 - abx^2$, and, if $x = \frac{2c}{a+b}$, shew that the product is equal to $-\frac{(a-b)^4}{(a+b)^4} c^4.$

3. Simplify $\frac{x^5 - 2x^4 - x^3 + 5x^2 - 4x + 1}{x^4 - x^3 - 3x^2 + 5x - 2}.$

4. A and B run a race; B has 50 yards start, but A runs 20 yards while B runs 19; what must be the length of the course that A may come in a yard ahead of B ?

5. There are four numbers proportional, whereof the third is the product of the first and second, whilst the fourth is the sum of the second and third, and the product of the first and third is equal to the sum of the second and fourth. Required the numbers.

6. Solve the equations:

$$(1) \sqrt{(2x+9)} - \sqrt{(x-4)} = \sqrt{(x+1)}.$$

$$(2) x^2 + xy + y^2 = 91; x + \sqrt{xy} + y = 13.$$

7. Multiply $x^{\frac{1}{2}} - y^{\frac{1}{2}}x^{-\frac{2}{15}} + 3y^{-\frac{1}{5}}x^{\frac{2}{3}}$ by $x^{\frac{1}{2}} - y^{\frac{1}{2}}$.

8. Square the expression $2\sqrt{5} - 3\sqrt{2} + \frac{1}{2}\sqrt{6}$.

9. There are 12 terms of an A. P. of which the first is $1\frac{1}{2}$ and the sixth is $-1\frac{1}{2}$; find their sum.

10. Shew that a simple equation can only have one root.

EXERCISE—LXXVIII.

1. Divide

$$apx^4 + x^3(aq + bp) + x^2(ar + bq + pc) + x(qc + br) + cr$$

by

$$ax^2 + bx + c.$$

2. Simplify $\frac{6x^3 - 13xy + 6y^2}{10x^3 - 9xy - 9y^2}$.

3. Solve the equations:

$$(1) \frac{6x+7}{9} + \frac{7x-13}{6x+3} = \frac{2x+4}{3}.$$

$$(2) \sqrt{(10+x)} - \sqrt{(10-x)} = 2.$$

4. Shew that if $\frac{cy+bx}{b-c} = \frac{ax+cx}{c-a} = \frac{bx+ay}{a-b}$, then

$$(a+b+c)(x+y+z) = ax+by+cz.$$

5. Shew that $(2^{n+4} - 2 \times 2^n) + (2^{n+2} \times 4) = \frac{7}{8}$.

6. Find the H. C. F. of $x^{\frac{1}{2}}y^{\frac{1}{2}} + xy + x + x^{\frac{1}{2}}yz^{\frac{1}{2}} + y^{\frac{1}{2}}z^{\frac{1}{2}} - 1$ and $x^{\frac{1}{2}}y^{\frac{1}{2}} + xy + x - x^{\frac{1}{2}}yz^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}} - 1$.

7. Simplify

$$\left\{ \frac{\sqrt{x}}{x^{\frac{1}{2}}-1} - \frac{1}{x^{\frac{1}{2}}+x+\sqrt{x}} \right\} + \left\{ \frac{x^{\frac{1}{2}}+1}{\sqrt{x}} + \frac{x^{\frac{1}{2}}-1}{\sqrt{x+1}} \right\}.$$

8. If $(a^2+b^2+c^2)(x^2+y^2+z^2) - (ax+by+cz)^2 = 0$, shew that $a : x = b : y = c : z$.

9. A vessel has two taps, *A* for letting off water, *B* for admitting it, and both taps being opened together, the vessel if full would be emptied in 6 hours. Now the vessel being already half-full, water is admitted by *B* for the time in which *A* could have emptied it, and then it contains as much as *A* could discharge in 100 minutes. Required the times in which the taps could by themselves respectively empty and fill the vessel.

10. Shew that a quadratic equation has two roots and only two.

EXERCISE—LXXIX.

1. Simplify $\left(x + \frac{1}{x} + 1\right) + \left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}} - 1\right)$.

2. Solve the equation

$$\frac{b-a}{x-b} - \frac{a-2b}{x+b} + \frac{3x(a-b)}{x^2-b^2} = 0.$$

3. Simplify

$$\left\{ (x^2y)^{\frac{1}{2}} z^{-\frac{1}{2}} \right\}^6 \text{ and } \left\{ \sqrt[n]{\left(x^{-\frac{1}{m+n}}\right)^n} \sqrt[n]{\left(x^{-\frac{1}{m+n}}\right)} \right\}^{-mn}$$

4. The pace of a tortoise upon a grass plot varies inversely as the length of the grass. If he can crawl 6 feet in a minute when the grass is an inch long, how long will it be when it takes him 20 minutes to travel 40 feet?

5. Simplify $\frac{\{a+b\sqrt{-1}\}^3 + \{a-b\sqrt{-1}\}^3}{\{a+b\sqrt{-1}\}^2 + \{a-b\sqrt{-1}\}^2}$.

6. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \dots$ prove that any one of these fractions is equal to $\frac{ax + \beta y + \gamma z + \dots}{aa + \beta b + \gamma c + \dots}$.

7. Extract the square root of

$$x^{\frac{8}{5}} - 2a^{-\frac{3}{5}}x^{\frac{11}{5}} + 2a^{\frac{4}{5}}x^{\frac{4}{5}} + a^{-\frac{6}{5}}x^{\frac{14}{5}} - 2a^{\frac{1}{5}}x^{\frac{7}{5}} + a^{\frac{8}{5}}.$$

8. If α and β be the roots of the equation $ax^2 + bx + c = 0$, prove that the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ will be $acx^2 + (2ac - b^2)x + ac = 0$.

9. If $x \propto \frac{z}{y^2}$ and $z \propto \frac{1}{x^2}$, shew that $x^3 \propto \frac{1}{y^2}$.

10. Shew that the equation $x^2 + mx + n = 0$ has its roots real and different, real and equal, or impossible, according as $m^2 > =$ or $< 4n$.

EXERCISE—LXXX.

1. Divide

$$x^3(a+1) - xy(x-y)(a+b) - y^3(b-1) \text{ by } x^2 - xy + y^2$$

2. Resolve $x^4 + 2xy(x^2 - y^2) - y^4$ into factors.

3. Divide $\frac{x^3}{x+1} - \frac{1}{x(x-1)}$ by $\frac{x^2+1}{x^2-1}$.

4. Solve the equations:

$$(1) \quad 3x + 2 - \frac{3x^2 + 2}{x^2 + 4} = 4.$$

$$(2) \quad \sqrt{5-x} + \sqrt{5+x} = \frac{12}{\sqrt{5+x}}.$$

$$(3) \quad (x+2y)^2 + 6y = 18 - 3x; \quad 3y + 2x = 5.$$

5. B , B_1 and B_2 are 3 bottles, of which B contains 2 quarts of water. If part of the contents of B be poured into B_1 so as to fill it, and the remainder into B_2 , it is found that B_2 is two-thirds full, but if it be first poured into B_2 so as to fill it, and the remainder into B_1 , it is found that B_1 is half full. Find how much B_1 and B_2 hold.

6. Extract the square root of $a^3 - x^3$ as far as the sixth power of x .

7. Shew that $x^4 + 2ax^3 + 3b^2x^2 - 4a^2x + 4b^4$ is a perfect square when $12ax = 7b^2 - a^2$.

8. If one-third of the sum of the squares of the roots of the equation $ax^2 + bx + c = 0$ be equal to their product, shew that $b^2 = 5ac$.

9. If $\frac{1}{b} + \frac{1}{c} = \frac{4}{a}$, shew that

$$(a+b-c)^2 + 2(b+c-a)^2 + (c+a-b)^2 = 2(b+c)^2.$$

10. Explain fully what is meant by the terms *Ratio* and *Proportion*.

EXERCISE—LXXXI.

1. Simplify :

$$(1) \frac{1+x-x^2+2x^3}{1+x^3} \text{ and } (2) \frac{a^2+2ab+b^2-c^2}{a^2+2ac-b^2+c^2}.$$

2. Solve the equations :

$$(1) \frac{x}{2} - \frac{2x-5}{3} + \frac{3x-10}{4} - \frac{4x-15}{5} + \frac{5x-20}{6} - \frac{6x-25}{7} = 0.$$

$$(2) \frac{y}{3} - \frac{x}{2} = 2; \frac{3-2x}{5} - \frac{4+5y}{11} = 4.$$

$$(3) \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2}; x^2 + y^2 = 90.$$

3. Divide $a+b+c+3a^{\frac{2}{3}}b^{\frac{1}{3}}+3a^{\frac{1}{3}}b^{\frac{2}{3}}$ by $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}$.

4. If $a:b=c:d$, shew that

$$a:b=(a^{\frac{1}{2}}+c^{\frac{1}{2}})^2:(b^{\frac{1}{2}}+d^{\frac{1}{2}})^2.$$

5. Find the coefficient of x^4 in the product of

$$1+\frac{x}{2}+\frac{x^2}{3}+\frac{x^3}{4}+\frac{x^4}{5}+\dots \text{ by } 1-\frac{x}{3}+\frac{x^2}{5}-\frac{x^3}{7}+\frac{x^4}{9}-\dots$$

6. A takes $n+2$ steps to $n+1$ of B 's, but $n-2$ of B 's steps are equal to $n-1$ of A 's. Which is the quicker walker?

7. The sum of n terms of an A. P., whose first term is a and constant difference $2ar$, being equal to n times the sum *ad infinitum* of the A. P., whose first term is a and constant factor r , prove that $r=\frac{n-2}{n-1}$.

8. A sets off from L to walk to M at the same time that B sets off from M to walk to L . When they meet A returns to L , and then walks to M , which he reaches at the same time that B reaches L . Compare their rates of walking.

9. What are the factors of $6x^3-7x^2-16x+12$?

10. Shew that a ratio of greater inequality is increased by subtracting the same number from both of its terms.

EXERCISE—LXXXII.

1. Simplify

$$x-\frac{1}{2}\left[\left(1\frac{1}{3}-x\right)-\frac{1}{3}\left(2\frac{1}{2}-x\right)-\frac{2}{3}\left\{1\frac{1}{2}-\left(x-2\frac{1}{4}\right)\right\}\right].$$

2. If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$, shew that

$$(b-c)x+(c-a)y+(a-b)z=0.$$

3. Find the H. C. F. of

$$x^4+4x^3-5x+2 \text{ and } x^5-x^4-3x^3+5x^2+3x-3.$$

4. Solve the equations:

$$(1) \frac{3x}{4x+1}+\frac{x}{4x-2}=1.$$

$$(2) 2(x+a)\{x+(x^2-a^2)^{\frac{1}{2}}\}=a^2.$$

5. If $x^3 + ax^2 + bx + c$ be exactly divisible by $x^2 + px + q$, shew that $p(p-a) = q-b$ and $q(a-p) = c$.

6. Sixty gallons of rum and water are sold at two-thirds of the price per gallon that the rum cost, and the merchant realizes a profit of 5 per cent. Find the quantity of water in the mixture.

7. If $\frac{p-q}{s-t} = \frac{q-r}{t-w}$ prove that either of these fractions is equal to $\frac{q(w-s) + rs - pw}{t(w-s)}$.

8. Extract the cube root of

$$x^3 + \frac{3x^{\frac{5}{2}}}{a^{\frac{1}{2}}} + 3x^2a^{\frac{1}{2}} + \frac{3x^3}{a} + 6x^{\frac{5}{2}} + \frac{x^{\frac{5}{2}}}{a^{\frac{3}{2}}} + 3ax + \frac{3x}{a^{\frac{1}{2}}} + 3a^{\frac{1}{2}}x^{\frac{1}{2}} + a^{\frac{3}{2}}.$$

9. If a, b, c, d be in geometric progression, shew that

$$(1) (a+b)^2 + (a-b-c+d)^2 + (c+d)^2 = 2(a+d)^2.$$

$$(2) (a+b)^2 + (a-b+d-c)^2 + (c+d)^2 = 2(a+d)^2 + 3(b+c)(a-c)(b-d).$$

10. If quantities be proportional according to the algebraical definition shew that they are proportional according to the geometrical definition.

EXERCISE—LXXXIII.

1. Prove that

$$(y-z)^2 + (z-x)^2 + (x-y)^2 = 3(y-z)(z-x)(x-y).$$

$$2. \text{ Simplify } (1) \frac{1}{x-3a} - \frac{1}{x+3a} + \frac{3}{x+a} - \frac{3}{x-a}.$$

$$(2) \frac{4a^3 - 5ab + b^3}{3a^3 - 3a^2b + ab^2 - b^3}.$$

3. Find the square root of

$$x^4 + \frac{5}{2}x^2 + \left(\frac{14}{3} + \frac{25}{16}\right)x^2 + \frac{35}{6}x + \frac{49}{9}.$$

4. Simplify (1) $6\sqrt{8} + \sqrt{50} - 7\sqrt{2} - 2\sqrt{18}$.

(2) $\sqrt[3]{40} - 5\sqrt{\frac{1}{25}}$.

5. Solve the equations:

(1) $\frac{3}{2}\sqrt{x} + 2\sqrt{x} = \frac{7}{\sqrt{x}}$.

(2) $x^2 + 12x - 365 = 2\sqrt{(x^2 + 12x - 5)}$.

(3) $(x+y)^2 + x + y = 12 = 2(x^2y^2 + xy)$.

6. A certain number consists of two digits, the difference between the digits is 4, and if 2 be added to the digit in the units' place multiplied by 5, the sum obtained will be equal to the number itself. Find the number.

7. Multiply $x^{-3} - x^{-2} + 4x^{-1} - 8$ by $x^{-3} + x^{-2} + 4x^{-1} + 8$.

8. If one arithmetic mean A , and two geometric means p and q be inserted between any two numbers, shew that

$$p^2 + q^2 = 2Apq.$$

9. What are eggs selling at, when, if they be raised three-pence the dozen, one would get four fewer in a shilling's worth?

10. If quantities be proportional according to the geometrical definition shew that they are proportional according to the algebraical definition.

EXERCISE—LXXXIV.

1. Divide $1 + x^3 + y^3 - 3yz$ by $1 + y + z$.

2. Find the H. C. F. of

$$x^4 - px^3 + (q-1)x^2 + px - q \text{ and } x^4 - qx^3 + (p-1)x^2 + qx - p.$$

3. Prove that $\frac{(1+xy)(1+xz)}{(x-y)(x-z)} + \text{two similar expressions}$ with the letters interchanged = 1.

4. Solve the equations :

$$(1) \sqrt{(a^2 + 2ax - 3x^2)} - \sqrt{(a^2 + ax - 6x^2)} \\ = \sqrt{\{(a + 3x)(2a - 3x)\}}.$$

$$(2) \left. \begin{aligned} \frac{3}{x} + \frac{4}{y} + \frac{1}{z} &= 4 \\ yz + zx + xy &= \frac{11}{6}xyz \\ 2xz + 3yz &= 2xy \end{aligned} \right\}.$$

$$(3) \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}; \quad \sqrt{\frac{x^2}{y}} + \sqrt{\frac{y^2}{x}} = \frac{9\sqrt{2}}{2}.$$

5. A cistern whose capacity is 960 gallons is filled in 30 minutes by 3 pipes, the first of which conveys 12 gallons more and the third 7 gallons less than the second. How much flows through each pipe in a minute ?

6. Multiply

$$x + \frac{1 - \sqrt{3} \cdot \sqrt{-1}}{2}y \text{ by } x + \frac{1 + \sqrt{3} \cdot \sqrt{-1}}{2}y.$$

7. Divide $x^3 + 3x^2/6 + 1$ by $x + \sqrt[3]{3} - \frac{2}{\sqrt[3]{2}}$.

8. Two numbers end with the same digit, and being divided by 7 the quotient of each is the remainder of the other ; the sum of these also is 7. What are the numbers ?

9. Sum (1) $a^p + a^{p+1} + a^{p+2} + \dots$ to n terms.

$$(2) \frac{2}{3} - \frac{1}{2} + \frac{3}{8} - \dots \text{ to infinity.}$$

10. Shew how to find the sum of a series of numbers in arithmetical progression.

EXERCISE—LXXXV.

1. Shew that

$$\frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} + \frac{(x-a)(x-b)}{(c-a)(c-b)} = 1.$$

2. Two vessels contain each a mixture of wine and water. In the first vessel the quantity of wine : quantity of

water = 1 : 3, and in the second = 3 : 5. What quantity must be taken from each in order to form a third mixture which shall contain 5 gallons of wine and 9 of water ?

3. Simplify

$$\frac{ab(a-b)(a^2+b^2)+bc(b-c)(b^2+c^2)+ca(c-a)(c^2+a^2)}{a^2b^2(a-b)+b^2c^2(b-c)+c^2a^2(c-a)}.$$

4. Solve the equations :

$$(1) \quad x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}} = 1\frac{1}{2}.$$

$$(2) \quad 5x+2 : 4y-1 = 4y+1 : 5x=3x+2y : x+3y.$$

5. What sum of money must a person invest at 5 per cent. per annum, so that after paying income-tax at the rate of 6*d.* in the pound he may have a clear annual income of £1170 a year ?

6. If α, β be the roots of the equation $ax^2+x+b=0$, shew that $\left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)=\frac{1}{ab}$.

7. If $a : b = c : d$, and a is the greatest of the four magnitudes, shew (1) that d is the least, (2) that $a+d$ is greater than $b+c$.

8. Divide $x+3x^{\frac{1}{2}}-2x^{\frac{1}{4}}-7x^{\frac{1}{8}}+2x^{-\frac{1}{4}}$ by $x^{\frac{1}{8}}-2x^{-\frac{1}{4}}$.

9. The population of a district increases from 100000 to 161051 in five years. Find the rate of increase per cent.

10. Shew how to find the sum of a series of numbers in geometrical progression.

PART III.

EXERCISE—LXXXVI.

1. Multiply

$$x^4 + 2ax^3 + 3a^2x^2 + 2a^3x + a^4 \text{ by } x^2 - 2ax + a^2.$$

2. Find the H. C. F. of

$$2x^3 - 16x + 6 \text{ and } 5x^3 + 15x^2 + 5x + 15.$$

3. Solve the equations :

$$(1) \ x + \frac{3x-5}{2} = 12 - \frac{2x-4}{3}.$$

$$(2) \ \frac{3x-4}{x-4} = 9 - \frac{x-2}{2}.$$

$$(3) \ \frac{10x+y}{xy} = 3; \ y - x = 2.$$

4. *A* and *B* began to pay off their debts. *A*'s money was at first two-thirds of *B*'s, but after *A* had paid £1 less than two-thirds of his money, and *B* £1 more than seven-eighths of his, it was found that *B* had only half as much as *A* had left. What sum had each at first?

5. Find the square root of

$$x^{\frac{3}{2}} + xy^{-\frac{1}{2}} - 2x^{\frac{5}{4}}y^{-\frac{1}{4}} - 2x^{\frac{1}{2}}y^{\frac{1}{4}} + 2x^{\frac{3}{4}}y^{\frac{1}{4}} + y.$$

6. If $\frac{1}{a}$, $\frac{1}{a} - \frac{1}{b}$, $\frac{1}{a} - \frac{1}{c}$ be in H. P., then will a , c , b be in A. P.

7. If the number of combinations of $2n$ things taken $n-1$ together : the number of combinations of $2(n-1)$ things taken n together = $132 : 35$, find n .

8. Given that the coefficient of the $(r+1)^{\text{th}}$ term of the expansion of $(1+x)^n$ is equal to that of the $(r+3)^{\text{th}}$ term, determine r .

9. Expand $\frac{1}{(1-3x)^3}$ to four terms.

10. If a, b, c be in harmonic progression, shew that

$$a : c = a - b : b - c.$$

EXERCISE—LXXXVII.

1. Simplify $\left\{1 + \frac{1}{\frac{x}{y} + 1}\right\} \left\{1 + \frac{1}{\frac{x}{y} - 1}\right\} - \frac{x(y+4x)}{4(x^2-y^2)}$.

2. Reduce $\frac{5+2\sqrt{3}}{3\sqrt{3}-5}$ and $\frac{(a+x)^{\frac{1}{2}} + (a-x)^{\frac{1}{2}}}{(a+x)^{\frac{1}{2}} - (a-x)^{\frac{1}{2}}}$ to fractions with rational denominators.

3. If a crew row 30 strokes a minute they can do the course in $8\frac{1}{2}$ minutes : if they row 40 strokes a minute they can do it in $7\frac{1}{2}$ minutes. Supposing the pace of the boat to vary as the product of the length of the stroke multiplied by the number of strokes per minute, and the length of each stroke to be diminished by a quantity proportional to the number over 30 rowed per minute, find how long they will be over the course if they row at the rate of 35 per minute.

4. Solve the equation $7x + \sqrt{(x^2 - 17x + 4)} = 2x^2 - 27x + 5$.

5. Find the square-root of $4ab + 2(a^2 - b^2)\sqrt{-1}$.

6. Shew that the sum of the roots of the equation

$$\frac{a'}{x-a} + \frac{b'}{x-b} + \frac{c'}{x-c} = 0$$

is equal to

$$\frac{a'(b+c) + b'(c+a) + c'(a+b)}{a' + b' + c'}$$

7. Shew that the coefficient of x^r in the expansion of $(1-4x)^{-\frac{1}{2}}$ is $\frac{|2r|}{(|r|)!}$.

8. If n be the first of two arithmetic means between two numbers, and ν the first of two harmonic means between the same two numbers, prove that the value of ν does not lie between the values of n and $9n$.

9. Given $\log 2 = \cdot 30103$ and $\log 3 = \cdot 47712$, find the logarithms of 72, $\sqrt[3]{\frac{1}{6}}$ and 135.

10. Shew that the arithmetic, geometric and harmonic means between two numbers are in descending order of magnitude.

EXERCISE—LXXXVIII.

1. Find the H. C. F. of $3x^5 + 10x^4 + 16x^3 + 27x^2 + 20x + 14$ and $6x^4 + 17x^3 + 19x^2 + 34x + 14$.

2. Solve the equations:

$$(1) \frac{x+4a+b}{x+a+b} + \frac{4x+a+2b}{x+a-b} = 5.$$

$$(2) \frac{\sqrt{(2a^2-x^2)} + b\sqrt{(2a-x)}}{\sqrt{(2a^2-x^2)} - b\sqrt{(2a-x)}} = \frac{\sqrt{a+b}}{\sqrt{a-b}}.$$

3. Find the values of a and b in order that $x^2 + 3xy + 4y^2$ may exactly divide

$$x^8 + 7x^5y + 6x^4y^2 + 5x^3y^3 + ax^2y^4 + bxy^5 + 12y^6.$$

4. The stakes at a regatta are equally divided among the crew of the winning boat in the following manner. The first receives £1 and a tenth of the remainder; the second £2 and a tenth of the remainder; the third £3 and a tenth of the remainder, and so on. Find the amount of the stakes and the number of the crew.

5. Apply the expansion of $(a+x)^{-n}$ to shew that

$$\sqrt{2} = \frac{7}{5} \left\{ 1 + \frac{1}{10^3} + \frac{1 \cdot 3}{1 \cdot 2} \cdot \frac{1}{10^4} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \cdot \frac{1}{10^5} + \dots \right\}.$$

6. If m be the number of combinations of n things taken 3 together, and the number of combinations of m things 2 together be equal to 17 times the number of combinations of n things 4 together, find m and n .

7. Transform 65·263 from the octenary scale to the senary scale.

8. If α, β be the roots of the equation $px^2+qx+q=0$, shew that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$.

9. Shew by mathematical induction that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}.$$

10. Find the number of permutations of n different things taken r at a time.

EXERCISE—LXXXIX.

1. Resolve $(a+b+c)(ab+bc+ca)-abc$ into factors.

2. Solve the equations:

$$(1) \quad x^3 + 4x + 2\sqrt{(x^2 + 4x + 11)} = 13.$$

$$(2) \quad \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{x} - \frac{1}{y} = 2\frac{3}{4}; \quad xy + 2 = 0.$$

3. A man finds that he can walk for 5 hours at the rate of 4 miles per hour, but that afterwards his speed in any hour varies inversely as the number of hours since he set out. If he traverse $3\frac{1}{2}$ miles in the seventh hour, find the whole distance traversed in 9 hours.

4. The first two terms of a series in harmonical progression are 3 and 4: continue the series to three more terms.

5. Shew that if $\rho x + my + nz = 0$ and $\rho'x + m'y + n'z = 0$, then $\frac{x}{mn' - m'n} = \frac{y}{n\rho' - n'\rho} = \frac{z}{\rho m' - \rho'm}$.

6. Prove that the ratio of the number of combinations of $4n$ things taken $2n$ at a time to that of $2n$ things taken n at a time is $\frac{1 \cdot 3 \cdot 5 \dots (4n-1)}{\{1 \cdot 3 \cdot 5 \dots (2n-1)\}^2}$.

7. Transform $1234 \cdot 1234$ from the scale of five to that of seven, so that the error may not exceed $\frac{1}{7^4}$.

8. Write down the $(r+1)^{\text{th}}$ term of $(1-2x)^{-\frac{1}{2}}$ and prove that it is equal to twice the coefficient of x^r in

$$\left(1 - \frac{1}{2}x\right)^{-r}.$$

9. Factorize the expression $15x^3 - 16x^2 - 17x + 14$.

10. Find the number of permutations of n things taken all together when certain of the things are alike.

EXERCISE—XC.

1. Simplify $\frac{(a+b)(1-ab)}{(1-ab)^2 - (a+b)^2} - \frac{a(1-b^2) + b(1-a^2)}{(1-a^2)(1-b^2) - 4ab}$.

2. Solve the equations:

$$(1) \quad x^4 + 5x - 6 = 5x^2(x-1).$$

$$(2) \quad x^3 + 11xy^2 = 30; \quad y(x^2 + y^2) = 5.$$

3. If a, b, c be in G.P., shew that $a+b, 2b, b+c$ are in H.P.

4. Write down the $(3r-1)^{\text{th}}$ term of $(2a-3x)^n$.

5. If the $(m+1)^{\text{th}}$ and $(m+2)^{\text{th}}$ coefficients in $(a+b)^n$ be equal, and also the $(m+3)^{\text{th}}$ and $(m+4)^{\text{th}}$ terms, shew that $(m+3)a = (m-1)b$.

6. If the m^{th} term of a H.P. be n , and the n^{th} term be m , shew that the r^{th} term will be $\frac{mn}{r}$.

7. A train in going from Cambridge to London stops at nine intermediate stations. Six persons enter the train during the journey with 6 different tickets. How many different sets of tickets may they have had?

8. Given $\log 2 = \cdot 30103$, $\log 3 = \cdot 47712$, find the logarithms of 2187, '0001, and $(\cdot 0025)^{\frac{1}{2}}$.

9. The number of dogs in a college equals $\frac{1}{3}$ the number of resident members. The number belonging to the undergraduates equals $\frac{3}{2}$ the number of dons, and the number belonging to the dons equals $\frac{1}{10}$ the number of undergraduates.

Compare the number of dogs and undergraduates.

10. Find the number of combinations of n things all different taken r at a time.

EXERCISE—XCI.

1. If $ax + by = 1 = a'x + b'y$, and $xy = \frac{1}{ab' + a'b}$, shew that

$$\frac{a}{a'} + \frac{a'}{a} + \frac{b}{b'} + \frac{b'}{b} = 4.$$

2. Solve the equations:

$$(1) \quad 6\sqrt{(x^2 - 2x + 6)} = 21 + 2x - x^2.$$

$$(2) \quad x^3 - y^3 = 513; \quad x - y = 3.$$

3. Shew that the r^{th} term of the expansion of $(1-x)^{-\frac{p}{q}}$ is
$$\frac{p \cdot (p+q) \cdot (p+2q) \cdots \{p + (r-2)q\}}{1 \cdot 2 \cdot 3 \cdots (r-1)} \cdot \left(\frac{x}{q}\right)^{r-1}.$$

4. Convert 297 from the common scale of notation to that of 11; then square the transformed number, extract the square root, and reconvert it to the common scale.

5. Find the greatest term of

$$\left(\frac{2x}{5} - \frac{x^2}{3}\right)^{-\frac{3}{2}} \quad \text{when } x = \frac{1}{10}.$$

6. The places of five men in an "Eleven" being determined, in how many ways could the field be arranged?

7. If the squares of two numbers between 20 and 30 end with the same digit, shew that they end with the same two digits.

8. If σ , s be the sums to infinity and to n terms of a geometrical series respectively, the first term of the series being a , shew that

$$n \log \left(1 - \frac{a}{\sigma} \right) = \log \left(1 - \frac{s}{\sigma} \right).$$

9. If a, b, c be in A. P., b, c, d in G. P., and c, d, e in H. P., prove that a, c, e are in G. P.

10. Shew that the number of combinations of n things taken r at a time is the same as the number taken $n-r$ at a time.

EXERCISE—XCII.

1. Divide

$$x^4 - a^4 + 2ax^3 + 2na^3x - (n^2 - 1)a^2x^2 \text{ by } x^2 + a^2 - (n-1)ax.$$

2. Find the H. C. F. of

$$1 + x^{\frac{1}{2}} + x + x^{\frac{3}{2}} \text{ and } 2x + 2x^{\frac{1}{2}} + 3x^2 + 3x^{\frac{3}{2}}.$$

3. Shew that

$$\frac{1}{a(a-b)(a-c)} + \frac{1}{b(b-c)(b-a)} + \frac{1}{c(c-a)(c-b)} = \frac{1}{abc},$$

and that

$$\frac{1}{x + \frac{1}{y + \frac{1}{z}}} \div \frac{1}{x + \frac{1}{y}} - \frac{1}{y(xyz + x + z)} = 1.$$

4. Simplify (1) $\frac{a^3 + (1+a)ab + b^3}{a^4 - b^4}.$

$$(2) \frac{e^{2x}x^3 + e^{2x} - x^2 - 1}{e^{2x}x^3 + 2e^x x^2 - e^{2x} - 2e^x + x^2 - 1}.$$

5. Solve the equations:

$$(1) \frac{x^2 - a}{bx} - \frac{a - x}{b} = \frac{2x}{b} - \frac{a}{x}.$$

$$(2) \frac{\sqrt{(a+\sqrt{x})}}{\sqrt[3]{x}} + \frac{\sqrt{(a-\sqrt{x})}}{\sqrt[3]{x}} = \sqrt[3]{x}.$$

$$(3) x(y+1) = \frac{2}{3}; \quad y(x+1) = \frac{1}{2}.$$

6. If a, b, c be in harmonic progression, shew that

$$\frac{b+a}{b-a} + \frac{b+c}{b-c} = 2.$$

7. A traveller sets out from a certain place and travels one mile the first day, two the second, and so on. In five days afterwards another sets out and travels 12 miles a day. How far must he travel to overtake the first?

8. Write down the r^{th} term of $(a-x)^{\frac{1}{2}}$.

9. Find a number of two digits which are transposed by either adding 18 to the number or by converting it into the septenary scale.

10. Find the value of r for which the number of combinations of n things taken r at a time is the greatest.

EXERCISE—XCIII.

1. Resolve into factors:

$$(1) a^2bx^2 + ab^2xy + acdxy + bcdy^2 - aefxz - bafyz.$$

$$(2) x^3 - 3x^2 - 193x + 195.$$

2. Solve the equations:

$$(1) \frac{x}{x^2-2x-15} - \frac{7.5}{x^2+2x-35} = \frac{1}{x^2+10x+21}.$$

$$(2) \frac{1}{x+a} + \frac{1}{x+2a} + \frac{1}{x+3a} = \frac{3}{x}.$$

$$(3) x^3 + 2xy + 3y^2 = 17; \quad 2x^2 + 3xy + 5y^2 = 28.$$

3. If $a : a_1 = b : b_1 = c : c_1$, and $A : A_1 = B : B_1 = C : C_1$,

$$\text{shew that } \frac{aC}{a_1C_1} = \left\{ \frac{(aA)^m + (bB)^m + (cC)^m}{(a_1A_1)^m + (b_1B_1)^m + (c_1C_1)^m} \right\}^{\frac{1}{m}}.$$

4. Find the greatest term in $(1-x)^{20}$ when $x = \frac{5}{12}$.
5. The combinations of n things 3 together are 24 times as many as the combinations of $\frac{n}{2}$ things 4 together. Find n .
6. If a, b, c be in A. P., a, β, γ in H. P., and $aa, b\beta, c\gamma$ in G. P., shew that $a : b : c = \frac{1}{\gamma} : \frac{1}{\beta} : \frac{1}{a}$.
7. Shew that any number of 6 digits formed by the repetition of three digits in the same order is divisible by 7, 11, and 13.
8. If a and β be the roots of $ax^2 + bx + c = 0$, shew that the equation whose roots are $\frac{a^3}{\beta^3}$ and $\frac{\beta^3}{a^3}$ is

$$a^3c^2x^2 + (5a^3bc^2 - 5ab^3c + b^5)x + a^2c^3 = 0.$$
9. Prove that $ax^2 + bx + c$ has the same sign as a , except when the values of x which make it vanish are real; and that when x is between those values the sign is the opposite of that of a .
10. Investigate the Binomial Theorem for a positive integral index.

EXERCISE—XCIV.

1. Two boats pass the lower end of a gang of barges at an interval of 2 minutes, and the upper end at an interval of $1\frac{1}{2}$ minutes, going up stream with given velocities v_1 and v_2 . How soon will the boats be together, assuming (1) that the barges are anchored, (2) that they are floating down stream with a velocity of 1 mile an hour?
2. Solve the equations :
 - (1) $\frac{x + \sqrt{12a - x}}{x - \sqrt{12a - x}} = \frac{\sqrt{a+1}}{\sqrt{a-1}}$.
 - (2) $(x^3 + 3x^2 + 34x + 37)^{\frac{1}{3}} - (x^3 - 3x^2 + 34x - 37)^{\frac{1}{3}} = 2$.
3. The number of combinations of $n+2$ things, 3 together.

is 11 times the number of combinations of $\frac{2n}{3}$ things, 2 together. Find n .

4. What is the middle term of the expansion of $(2a-5b)^8$?

5. If a men or b boys can dig m acres of ground in n days, shew that the number of boys whose assistance will be required to enable $a-p$ men to dig $m+p$ acres in $n-p$ days will be $\frac{pb}{a} \left(1 + \frac{n+m}{n-p} \cdot \frac{a}{m} \right)$.

6. If in the scale whose radix is 12 a square number ends with a single cipher the preceding digit is 3, and the cube of the square root ends with 60.

7. Of four quantities a, b, c, d , if a, b, c be in A. P. and b, c, d be in H. P., then $a:b=c:d$, and if a, c, d be also in A. P., a, b, d are in H. P.

8. Prove that the n^{th} terms in the expansions of $(1-x)^{-n}$ and $(1+x)^{2n-2}$ are equal.

9. If two series in A. P. have the same sum but not the same first term, shew that they must have the same last term.

10. Find the r^{th} term of the expansion of $(a+x)^n$, where n is positive and integral.

EXERCISE—XCV.

1. There is a series of numbers in A. P. beginning from 1, in which the sum of the first half of any even number of terms bears to the sum of the second half the same constant ratio R . Find the constant difference.

2. The contents of a coin vary as the thickness and as the area of the face, and this area varies as the square of its radius: now supposing that when a shilling is as thick as a sovereign it is 20 per cent. broader, compare the thicknesses of a half-crown and a five-pound piece, which is of the same breadth as the half-crown.

3. Find the coefficient of x^3 in the expansion of $(1+x)^{\frac{1}{3}}$.

4. If a, b, c be in H. P., shew that

$$\frac{1}{b-c} + \frac{4}{c-a} + \frac{1}{a-b} = \frac{1}{c} - \frac{1}{a}.$$

5. Expand $\left(x - \frac{1}{x}\right)$ to four terms.

6. Solve the equations:

$$(1) \frac{1}{21x^3 - 13x + 2} + \frac{1}{28x^3 - 15x + 2} = 12x^3 - 7x + 1.$$

$$(2) x^3 + y^3 = \frac{1}{2} + x; x^3(x-2) + 6y^2x(x-1) + y^4 = \frac{1}{4} - x.$$

7. Prove by induction that the first two terms of the expansion of $(a+b)^n$, where n is a positive integer, are $a^n + na^{n-1}b$.

8. If $x:y$ be the ratio $a:b$ in its lowest terms, prove that the ratio $x+1:y+1$ is greater than the ratio $a+1:b+1$.

9. A and B go round the world, of which the circuit is 23661 miles. A goes east one mile the first day, two the second, and so on in arithmetic progression. B goes west at the uniform rate of twenty miles a day. Find when they will meet, and interpret the negative answer.

10. Shew that the coefficient of the r^{th} term from the beginning of the expansion of $(a+x)^n$ is identical with the coefficient of the r^{th} term from the end.

EXERCISE XCVI.

1. Resolve into elementary factors:

$$(1) (1+a)^3(1+b^3) - (1+b)^2(1+a^2).$$

$$(2) x^4 - (a+b)x^3 + (a^2b + ab^2)x - a^2b^2.$$

2. If $A = a^2 - bc$, $B = b^2 - ca$, $C = c^2 - ab$, prove that

$$\frac{A^2 - BC}{a} = \frac{B^2 - CA}{b} = \frac{C^2 - AB}{c} = (a+b+c)(A+B+C).$$

3. Solve the following equations:

$$(1) \frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-c)(x-a)}{(b-c)(b-a)} = x.$$

$$(2) x^6 + (x+1)^6 + 1 = 2(x^3 + x + 1)^2.$$

4. If m, n be any two numbers, g their geometric mean, a_1, h_1 and a_2, h_2 the arithmetic and harmonic means between m, g and g, n respectively, shew that $a_1 h_1 = g^2 = a_2 h_2$.

5. A certain multiple of 20 has three digits when it is expressed either in the scale of 4 or the scale of 5, and the corresponding digits in the two cases are in the ratio of 3 : 2. Find the number.

6. What term of $(1-x)^{\frac{1}{2}}$ is equal to $\frac{1}{13}$ of the same term of $(1+x)^{-\frac{1}{2}}$?

7. If $C_1, C_2, C_3, \dots, C_n$ be the number of combinations of n things taken 1, 2, 3, ..., n at a time; prove that

$$C_1 + C_2 + C_3 + \dots + C_n = 2^n - 1.$$

8. If S_n represents the sum of n terms of an A.P. then $S_{n+5} - 2S_{n+6} + S_{n+7} = d$, where d is the constant difference of the progression.

9. Prove that

$$\frac{a^4(b^3 - c^3) + b^4(c^3 - a^3) + c^4(a^3 - b^3)}{a^3(b-c) + b^3(c-a) + c^3(a-b)} = (b+c)(c+a)(a+b).$$

10. Find the greatest term of the expansion of $(a+x)^n$ when n is a positive integer.

EXERCISE—XCVII.

1. Find the L.C.M. of

$$x^5 - 2x^4 - 6x^3 + 4x^2 + 13x + 6 \text{ and } 3x^4 + 4x^3 - 6x^2 - 12x - 5.$$

2. Extract the cube root of

$$1 + 9x + 9x^2 - 51x^3 + 126x^4 + 234x^5 - 996x^6 + 1980x^7 - 1800x^8 + 1000x^9.$$

3. A starts at noon to travel from P to Q at the rate of 6 miles an hour; B starts at 6 h. 30 m. P.M. from Q at the rate of 4 miles an hour, and subsequently C at the rate of 5 miles an hour to overtake B : when this happens C is within 1430 yards of having walked 3 times as far, and B has walked twice as far as they had when A met them. When did C leave Q ?

4. Solve the equations:

$$(1) \quad x + (x^2 - a^2)^{\frac{1}{2}} = \frac{8}{\sqrt{3}}.$$

$$(2) \quad x + y = 1; \quad x^5 + y^5 = 31.$$

5. In how many ways may the letters of the word *Denomination* be written?

6. Write down the r^{th} term of $(a^{\frac{1}{2}} - x^{\frac{1}{2}})^{\frac{10}{3}}$.

7. Shew that 961 is a square number in any scale of notation.

8. Transform 1.69 to the scale of four, extract its square root in that scale, and reduce the root to the scale of ten.

9. In $(1 - 2x - 2x^2)^{\frac{7}{2}}$ find the coefficient of x^4 .

10. Shew that the sum of all the coefficients in the expansion of $(a+x)^n = 2^n$.

EXERCISE—XCVIII.

1. If a, b, c be in H. P., shew that

$$\frac{1}{a} + \frac{1}{b+c}, \quad \frac{1}{b} + \frac{1}{a+c}, \quad \text{and} \quad \frac{1}{c} + \frac{1}{a+b}$$

will also be in H. P.

2. Expand $\frac{1}{(1-x^2)^{\frac{1}{2}}}$ to five terms.

3. If $x = 1 + h$, where h is very small, shew that

$$\sqrt{\left(\frac{2x - x^2 + a^2 x^2}{a^2 + 1}\right)} = 1 + \frac{a^2 h}{a^2 + 1} \text{ nearly.}$$

H. S. Ex.

4. Eight boats start in a race one after another. Find the number of ways in which they can be arranged so that
 (1) a particular boat may always have a given boat behind it,
 (2) that it may always be between two given boats.

5. Solve the equations :

$$(1) \quad 4x^3 + 6x^2 + x = 1.$$

$$(2) \quad x + \sqrt{(x+y)} = 12 - y; \quad x^2 + y^2 = 41.$$

6. Transform 1860 from the denary to the septenary scale

7. Find the 10th term of $\left(\frac{5}{2}x^3 - \frac{2}{5}y^3\right)^{-\frac{2}{3}}$

8. The number 456·1 is in the scale of 7; bring it to the scale of 14, and prove that your result is correct by returning to the original scale.

9. The roots of the quadratic $x^2 + ax + b = 0$ are α and β . Form the quadratics whose roots are α and $-\beta$, β and $-\alpha$ respectively, and verify by comparing them with the biquadratic whose roots are $\pm \alpha$ and $\pm \beta$.

10. Shew that the sum of the coefficients of the odd terms in the expansion of $(a+x)^n$ is equal to the sum of the coefficients of the even terms.

EXERCISE—XCIX.

1. If $x^2 + a_1x + b_1$ and $x^2 + a_2x + b_2$ have a common factor of the form $x + a$, shew that $a = \frac{a_2b_1 - a_1b_2}{b_1 - b_2}$.

2. Find the H. C. F. of

$$ab + 2a^3 - 3b^3 - 4bc - ac - c^3 \text{ and } 9ac + 2a^3 - 5ab + 4c^3 + 8bc - 12b^3.$$

3. Solve the equations:

$$(1) \quad \frac{x^3 + 1}{x + \sqrt{3}} = 2x - 4\sqrt{2}.$$

$$(2) \quad x^4 + x^2y^2 + y^4 = 91; \quad 4(x^2 - xy + y^2) = \frac{52}{(x-y)^2}.$$

4. A drawer is fitted with n compartments, and each compartment contains n counters, no two of which are marked alike. Prove that the total number of combinations which can be made with these counters, when no two out of the same compartment enter into any combination, is $(n+1)^n - 1$.

5. If the number $r^3 - r - 3$ be transformed to the scale whose radix is r , shew that the digits of the number so obtained will be $r-1, r-2, r-3$.

6. What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 5 or 6 deep, and also of its being formed into a solid square?

7. If α and β be the roots of the equation

$$x^3 + rx + \frac{3}{16}r^2 = 0,$$

determine the equation whose roots are $\alpha^3 + \beta^3$ and $\alpha^3 - \beta^3$.

8. Prove that $a_2a_3 - a_1a_4$ is positive, zero, or negative, according as a_1, a_2, a_3, a_4 are in arithmetical, geometrical or harmonical progression.

9. Write down the n^{th} term in the expansion of

$$\left(n - \frac{x}{\sqrt{2}}\right)^{\frac{1}{n}}.$$

10. Give Euler's proof of the Binomial Theorem for fractional and negative indices.

EXERCISE—C.

1. If $x = cy + bz$, $y = az + cx$, $z = bx + ay$, shew that

$$\frac{x^2}{1-a^2} = \frac{y^2}{1-b^2} = \frac{z^2}{1-c^2}.$$

2. A person walking at a given rate (a) along a road parallel to a railway is overtaken and met by two trains travelling at given rates (r_1 and r_2), and he observes that the time each takes in passing him is the same. Compare the lengths of the trains.

3. Write down the $(r+1)^{\text{th}}$ term of $(1-2x)^{-\frac{1}{2}}$ and prove that it is equal to twice the coefficient of x^r in $\left(1-\frac{1}{2}x\right)^{-r}$.

4. A and B are in a company of 72 men, which is divided into squads of 6. In how many of the different possible arrangements will it happen that A and B are in the same squad?

5. If a be rational and b a quadratic surd, prove that if $a^2 - b^2 = c^2$,

$$\sqrt{a+b} = \sqrt{\left(\frac{a+c}{2}\right)} + \sqrt{\left(\frac{a-c}{2}\right)}.$$

6. The number of combinations of n things taken three together : the number of permutations of $\frac{n}{2}$ things taken 3 together = 7 : 3. Find n .

7. Three quantities a, b, c are (1) in arithmetical, (2) in geometrical, (3) in harmonical progression. Find the value of $\frac{a(b-c)}{a-b}$ in each case.

8. What is the condition that the three equations, $x-ay=b$, $x-a_1y=b_1$, $x-a_2y=b_2$ may be satisfied by the same values of x and y ?

9. The logarithm of a number to the base 8 is 2.6532, what is its logarithm to the base 4?

10. Shew how to transform an integral number from one scale of notation to another.

EXERCISE—CI.

1. A and B run a race round a two-mile course. In the first heat B reaches the winning-post 2 minutes before A . In the second heat A increases his speed 2 miles an hour, and B diminishes his by the same quantity; and A then arrives at the winning-post 2 minutes before B . Find at what rate each ran in the first heat.

2. The number of combinations of n things taken 4 together : the number of combinations of $n-1$ things taken 3 together = 7 : 4. Find n .

3. Solve the equations :

$$(1) \quad x^2y - y^2x + 20(x^2 + y^2) = 0; \quad y^2 - 20x^2 = xy.$$

$$(2) \quad x + y = 72; \quad x^{\frac{1}{3}} + y^{\frac{1}{3}} = 6.$$

4. Expand $\frac{1}{(1-x)^6}$ to 4 terms and give its general term.

5. Three numbers are in G.P.: but if each be increased by 15 they will be in H.P., and the sum of the numbers is 49. Find them.

6. Express 142 in the scale whose radix is 7. Change the decimal equivalent to three-eighths into a radical fraction in this scale.

7. If the ratio of the difference of the antecedents of two ratios to the difference of the consequents is measured by the sum of the measures of the separate ratios, the antecedents are in the duplicate ratio of the consequents.

8. There are 10 soldiers and 8 sailors. How many different parties of 6 can be made, each party consisting of 3 soldiers and 3 sailors?

9. What are the characteristics of the logarithms of 626 to the bases 3, 4, 5?

If $\log_3 3 = a$, express $\sqrt{27}$.

10. Shew that in any scale whose radix is r the sum of the digits of an integer divided by $r-1$ will leave the same remainder as the integer leaves when divided by $r-1$.

EXERCISE—CII.

1. Find the H.C.F. of $5x^2y - 3x^4y^3 - 43x^2y^3 + 27x^2y^4 - 18xy^4$ and $7x^6y^3 - 2x^5y^3 - 63x^4y^4 + 18x^2y^3$.

2. Solve the equations:

$$(1) \sqrt{a+x} - \sqrt{a-x} = \sqrt{x}.$$

$$(2) x+y^2=y+x^2; \quad xy=\frac{3}{16}.$$

3. A wine merchant is selling two wines A and B at the same price, x shillings per dozen, and is making a profit of 10 per cent. on A . If a customer will take 20 dozen of A and 30 dozen of B , the wine merchant will reduce the price of each by 5 per cent., and will still realize the same rate of profit. What profit does he make by selling B at x shillings a dozen?

4. Write down the 7th term of $(1-5x)^{-\frac{1}{2}}$.

5. The sum of six terms of the series $1-x\sqrt{-1}-x^2+\dots$ is equal to 65 times the sum to infinity. Find x .

6. Shew that the number of combinations of $2n$ things, n of which are alike, taken n together $= 2^n$.

7. If s be the sum of any number of terms of the series of natural numbers beginning from 1, shew that $8s+1$ is a square.

8. Given $\log 2 = .301300$ and $\log 3 = .477121$, find

$$\log(72)^{\frac{1}{2}} \text{ and } \log(.0072)^{\frac{1}{2}}.$$

9. If α, β be the roots of $x^2+px+q=0$, and α, γ those of $x^2+rx+s=0$, prove that $(\beta+\gamma)$ and $\beta\gamma$ respectively satisfy the equations

$$x^2+(p+r)x+2(q+s)=0 \text{ and } x^2+(q+s-pr)x+qs=0.$$

10. Shew that in any scale a radical fraction is a proper fraction.

EXERCISE—CIII.

1. A and B run round a field, starting from the same point in opposite directions. They reach the starting-point

4 and 9 minutes respectively after they meet. If they continue to run at the same rate, in what time will they meet at the starting-point?

2. Prove that in any scale whose radix is r , a number consisting of more than r digits must have at least two digits alike.

3. Expand $(1-x^{\frac{1}{2}})^{\frac{1}{2}}$ to five terms.

4. Solve the equations:

$$(1) \quad (x+1)\left(1+\frac{1}{x^2+1}\right) + \sqrt{2x}\left(1+\frac{1}{x^2+1}\right) = 2.$$

$$(2) \quad x^3 + y^3 = 152; \quad x^2y + xy^2 = 120.$$

5. Given that the coefficient of the $(r+1)^{\text{th}}$ term of the expansion of $(1+x)^n$ is equal to that of the $(r+3)^{\text{th}}$ term, determine r .

6. A gentleman invites a party of $m+n$ friends to dinner, and places m at one table and n at another, the tables being round. Find the number of ways in which he can arrange the guests.

7. If the roots of $x^2+px+q=0$ and $x^2+qx+p=0$ differ by the same quantity, shew that $p+q+4=0$.

8. There are three numbers in arithmetical progression; the square of the first added to the product of the other two is 16; the square of the second added to the product of the other two is 14. Find the numbers.

9. Prove the following equalities:

$$(1) \quad a^3(b+c)^2 + b^3(c+a)^2 + a^3(b+c)^2 + 2abc(a+b+c) \\ = 2(ab+bc+ca)^2.$$

$$(2) \quad a^2b^2c^2 - a^2(s-a)^4 - b^2(s-b)^4 - c^2(s-c)^4 \\ + 2(s-a)^2(s-b)^2(s-c)^2 \\ = 2\{(s-b)(s-c) + (s-c)(s-a) + (s-a)(s-b)\}^2.$$

if $2s = a+b+c$.

10. Shew how to transform a fraction expressed in a given scale into a radical fraction in any other scale.

EXERCISE—CIV.

1. If $2s = a + b + c$, shew that

$$2(s-a)(s-b)(s-c) + a(s-b)(s-c) + b(s-c)(s-a) + c(s-a)(s-b) = abc.$$

2. Find the square root of

$$4 \left\{ \sqrt[3]{\frac{a^3}{c^3}} - \sqrt[3]{\frac{a}{c}} + \sqrt[3]{\frac{c^3}{a^3}} - \sqrt[3]{\frac{c}{a}} \right\} + 9.$$

3. Three competitors A , B , C enter for a mile race. A , who can run the distance in $4\frac{1}{2}$ minutes, gives B a start of 20 yards and C a start of 60 yards. A reaches the winning-post two seconds before C , and C two seconds before B . When and where will A overtake B and C ?

4. If $(a^3 - 4b)^{\frac{1}{3}}$ be rational, prove that the difference between the roots of the equation $x^2 + ax + b = 0$ is also rational.

5. The sum of a geometrical series to infinity $= \frac{2}{3+4x}$.

Find the series and its sum to n terms.

6. Prove by means of the Binomial Theorem that

$$\left(x + \frac{1}{x}\right)^6 - \left(x - \frac{1}{x}\right)^6 = 12 \left(x^4 + \frac{1}{x^4} + \frac{10}{3}\right).$$

7. If N be the greatest integer which can be expressed by n digits in a certain scale, prove that the greatest integer of $2n$ digits in the same scale is $N^2 + 2N$.

8. Write down the middle terms of $(a+x)^{2n+1}$, n being integral and positive.

9. How many different sides can be formed in a croquet party consisting of 5 ladies and 3 gentlemen, the gentlemen never being all on the same side?

10. Define the terms Logarithm, Mantissa, Characteristic.

EXERCISE—CV.

1. Find the L. C. M. of

$$x^3 + 3x^2 - 6x - 8, \quad x^3 - 2x^2 - x + 2, \quad \text{and} \quad x^3 + x - 6.$$

2. A man can walk 45 miles in 9 hours when the wind is with him, in 15 hours when it is against him, and in 10 hours when it is calm. The wind adds or takes away from his rate of walking a quantity proportional to the square of the rate at which it passes him. Find the velocity of the wind.

3. Solve the equations:

$$(1) \quad x + y - \sqrt{xy} = 7; \quad x^2 + y^2 + xy = 133.$$

$$(2) \quad x + \sqrt{x} + \sqrt{x+2} + \sqrt{x^2 + 2x} = b.$$

4. If between any two given numbers there be inserted two arithmetical means A_1, A_2 , two geometrical means G_1, G_2 , and two harmonical means H_1, H_2 , shew that

$$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2}.$$

5. Find the number of combinations of the letters in the word *Proportion* taken 3 at a time.

6. Shew that when n is even the coefficient of the middle term of $(a+x)^n$ is

$$\frac{1 \cdot 3 \cdot 5 \dots (n-1)}{1 \cdot 2 \dots \frac{n}{2}} \cdot \frac{2^n}{2^{\frac{n}{2}}}.$$

7. Write down the 5th term in the expansion of $(2+x)^7$ and find the greatest term when $x=3$.

8. If p_r = coefficient of x^r in the expansion of $(1+x)^n$, n being a positive integer, prove that

$$\frac{p_1}{p_0} + 2 \frac{p_2}{p_1} + 3 \frac{p_3}{p_2} + \dots + \frac{n p_n}{p_{n-1}} = \frac{n(n+1)}{2}.$$

9. Shew that

$$\left(\frac{a+x}{a-x}\right)^{\frac{1}{2}} = 1 + \frac{x}{a+x} + \frac{3}{2}\left(\frac{x}{a+x}\right)^2 + \frac{5}{2}\left(\frac{x}{a+x}\right)^3 + \dots$$

10. Shew that $\log_a mn = \log_a m + \log_a n$.

EXERCISE—CVI.

1. If $\frac{bx+ay-cx}{a^2+b^2} = \frac{cy+bz-ax}{b^2+c^2} = \frac{az+cx-by}{c^2+a^2}$, shew that

$$\frac{x+y+z}{a+b+c} = \frac{ax+by+cz}{ab+bc+ca}.$$

2. Solve the equation

$$\sqrt{(x^2-4x+12)} - \frac{x^2+1}{2} = 4-2x.$$

3. Suppose 5 candidates are examined for 2 scholarships, and that A obtains $\frac{2}{5}$ ths of the whole number of marks given,

B twice as many as A gets more than C , who obtains 3 times as many as B gets more than D ; that D obtains half as many as A, B, C together, and E one-third more than the excess of the sum of A, B , and C 's marks together over D 's. Determine the successful candidates.

4. If the harmonic means between each pair of the quantities a, b, c be in A.P., then a^2, b^2, c^2 will be in H.P.; and if the harmonic means be in H.P., a, b, c will be in H.P.

5. Find the number of sequences of r cards that can be made out of a pack of n suits, each suit containing p cards.

6. Shew that the $(r+1)^{\text{th}}$ term of the expansion of $(1-4x)^{-\frac{1}{2}}$ is $\frac{1 \cdot 3 \cdot 5 \dots (2r-1)}{r!} \cdot (2x)^r$.

7. If S_p denote the sum of the series $a+ar^p+ar^{2p}+\dots$ to infinity, and S_{-p} the sum of the series $a-ar^p+ar^{2p}-\dots$ to infinity, then

$$(1) S_2 \cdot S_{-2} = aS_4,$$

$$\text{and } (2) \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \dots + \frac{1}{S_n} = \frac{n}{a} - \frac{r}{a} \cdot \frac{r^n-1}{r-1}.$$

8. Prove that the sum of the squares of the coefficients of the expansion of $(1+x)^n$ is

$$\frac{2n(2n-1) \dots (n+1)}{1 \cdot 2 \dots n}.$$

9. Shew that $\frac{c+2a}{c-b} + \frac{c+2b}{c-a} = 4, > 7, \text{ or } > 10$ according as c is the arithmetic, geometric or harmonic mean between a and b .

10. Shew that $\log_a \frac{m}{n} = \log_a m - \log_a n$.

EXERCISE—CVII.

1. Resolve $n^4 - 4n^3 + 5n^2 - 2n$ into its elementary factors, and hence shew that it is divisible by 12 for all integral values of n above 2.

2. If N be a number ending in 5, prove that the last two digits of N^2 are 25, and that the remainder forms the product of two consecutive numbers.

3. There are $2n$ guests at a dinner-party. Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must not be placed next to one another, find the number of ways in which the company can be placed.

4. Prove that no coefficient in the expansion of $(1-x)^{-m}$ can be equal to that which follows it next but one, unless all the coefficients are equal.

5. If a, b, c be in A. P., α, β, γ in H. P., and

$$\frac{a}{\gamma} + \frac{\gamma}{a} = \frac{\alpha}{c} + \frac{c}{\alpha},$$

shew that $a\alpha, b\beta$, and $c\gamma$ are in G. P.

6. Solve the equations:

$$(1) \quad x^4 - 2x^3 + x = 132.$$

$$(2) \quad y^3 - 64 = 8y\sqrt{x}; \quad y - 4 = 2\sqrt{xy}.$$

7. If $\frac{x}{y+z} = a$, $\frac{y}{x+z} = b$, $\frac{z}{x+y} = c$, shew that

$$x^2 : y^2 : z^2 = a(1-bc) : b(1-ca) : c(1-ab).$$
8. If a, b, c are the p^{th} , q^{th} and r^{th} terms of an A. P. and also of a G. P., shew that

$$(b-c) \log a + (c-a) \log b + (a-b) \log c = 0.$$
9. Find the r^{th} term of the expansion $\frac{1}{(a^5 - x^5)^{\frac{1}{5}}}$.
10. Shew that $\log_a m^r = r \log_a m$.

EXERCISE—CVIII.

1. Shew that $x^2 + qx + 1$ and $x^3 + px^2 + qx + 1$ have a common factor of the form $x + a$ when $(p-1)^2 - q(p-1) + 1 = 0$.

2. Solve the equations:

$$(1) \quad \sqrt{5 + \sqrt{x}} + \sqrt{5 - \sqrt{x}} = \frac{6}{\sqrt{5 + \sqrt{x}}}.$$

$$(2) \quad \frac{x+6}{x-8} - \frac{x-8}{x+6} = \frac{7(x-1)}{x^2 - 2x - 72}.$$

$$(3) \quad 2\sqrt{6\sqrt{x} + 6\sqrt{y}} + \sqrt{x} + \sqrt{y} = 18; \quad x - y = 12.$$

3. The rates of walking of A, B, C, D are in proportion, D 's rate being the least. A and B start together from a place F to meet D and C , who start at the same time from a place G . Shew that A will meet D before B meets C .

4. Shew that

$$\left(\frac{1+x}{1-x}\right)^n = 1 + n \frac{2x}{1+x} + \frac{n \cdot (n+1)}{1 \cdot 2} \cdot \frac{2^2 \cdot x^2}{(1+x)^2} + \dots$$

5. A given series consists of n numbers; the first term is a number of one digit, the second a number of two digits, and so on, and all the digits are the same: find the sum of such a series in any scale.

6. The number of combinations of n things r together equals p times the number in which any specified thing occurs, and equals q times the number in which any two specified things occur. Find n and r .

7. Shew that the coefficient of x^n in the expansion of $\frac{(1-2x)^2}{(1-x)^4}$ is $\frac{1}{6}(n-6)(n^2-1)$.

8. The continued product of four consecutive numbers is 1680; find them.

9. Shew that 12321 is a square number, whatever be the radix of the scale.

10. If x and y be the logarithms of m to the bases a and b respectively, shew that $y = \frac{1}{\log_a b} x$.

EXERCISE—CIX.

1. Resolve $x^3(y-z) + y^3(z-x) + z^3(x-y)$ into factors.

2. Solve the equations:

$$(1) \quad (x+4)(x-3) = 36 - \sqrt{\{(x+3)(x-2)\}}.$$

$$(2) \quad x - \frac{2}{\sqrt{x}} = 3.$$

3. A boat's crew row over a course $\frac{5}{4}$ miles in length against a stream, which flows at the rate of two miles an hour, in ten minutes. The usual rate of the stream is $\frac{1}{2}$ a mile per hour. Find the time which the boat would take in the usual state of the stream.

4. If a, b, c be in geometric progression, and p, q the arithmetic means between a, b and b, c respectively, then b will be the harmonic mean between p and q .

5. Expand $\{x^2(1-x^2)\}^{\frac{1}{2}}$ to four terms.

6. Express 957 in the septenary scale.

7. Find the number of combinations that can be made out of the letters in the following line:

ἀπαπαπαῖ, παπαπαπαπαπαπαπαπαῖ (Soph. *Philoct.* 746),

taking them (1) 5 at a time, (2) 25 at a time.

8. If the equations $x^2 - 9x + a = 0$, $x^2 - 14x + 3a = 0$ have a common root, solve both of them.

9. If a pin be put into the Great Eastern on one day and the number of pins doubled on each successive day, on what day will the vessel sink if it can only carry 22000 tons, and 256 pins weigh an ounce?

10. Prove that $\log_a b \times \log_b a = 1$.

EXERCISE—CX.

1. If $a - x : a - y : a - z = px : qy : rz$, and p, q, r be in arithmetical progression, shew that x, y, z are in harmonical progression.

2. Two men, A and B , are engaged to do two equal pieces of work. A begins one piece and B the other, and after working for 4 days they change pieces. A finishes his piece in 13 days and then returns and helps B , and after working together for 3 days they finish his piece. Shew that A is twice as good a workman as B .

3. Sum $x^{\frac{1}{2}} - ax + \frac{a^2}{\sqrt{x}} - \dots$ to n terms.

4. Write down the first four terms and the r^{th} term of the expansion of $(a^2 - x^3)^{-\frac{1}{2}}$.

5. Find the number of different ways in which 10 men may be drawn up in double rank, supposing 3 particular men always to be in the front and 3 others in the rear.

6. Express 23:32 in the quaternary scale.

7. The time required to walk from A to B at a uniform rate of $3\frac{1}{2}$ miles an hour is 4 minutes less than that required to walk half the distance at 3 miles an hour, and the other half at $3\frac{3}{4}$ miles an hour; what is the distance?

8. If a_r and b_r be the arithmetical and geometrical means between a_{r-1} and b_{r-1} , prove that a_{n-1} and b_{n-1} are equal to

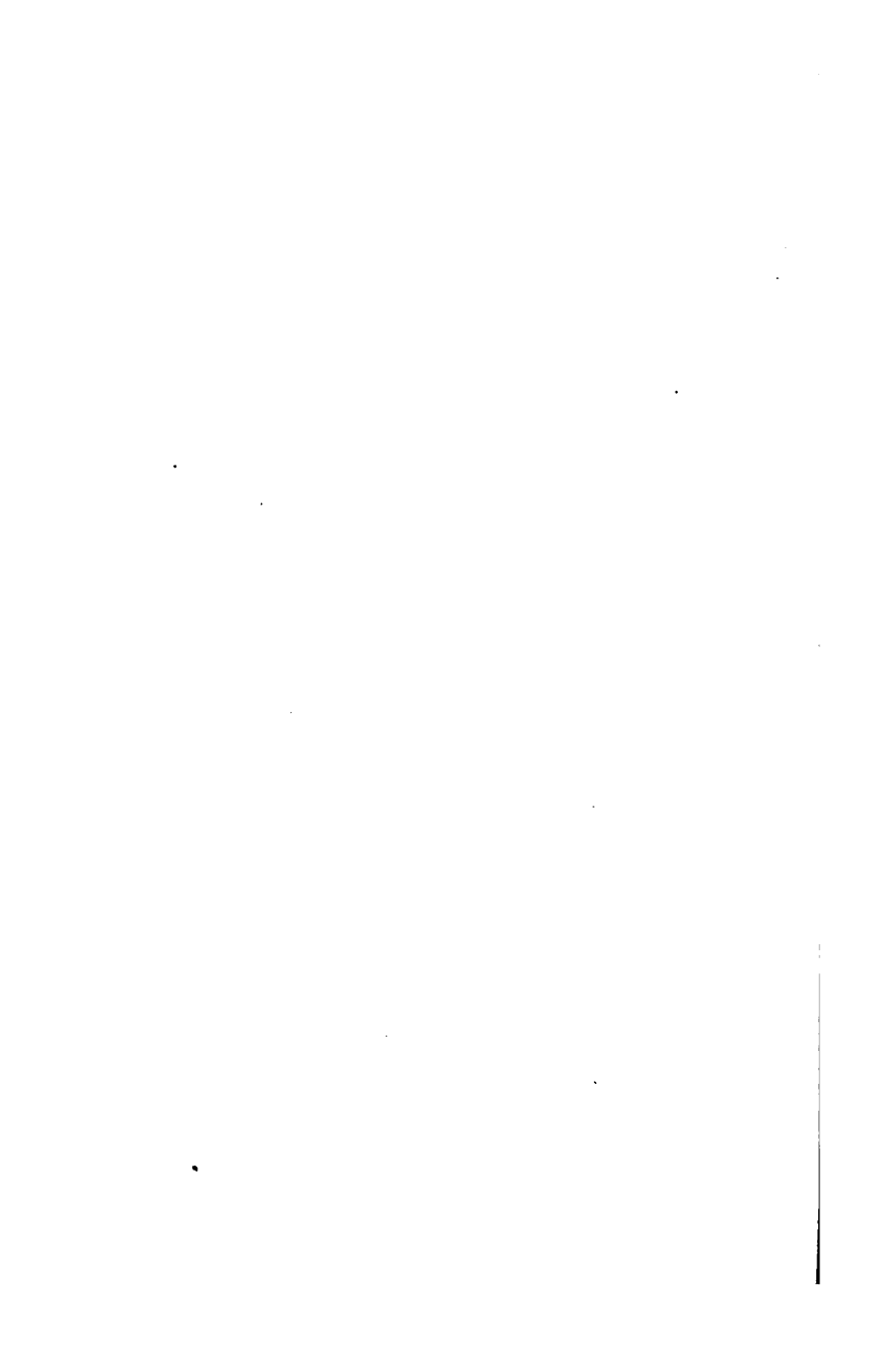
$$\{a_n^{\frac{1}{2}} \pm (a_n^2 - b_n^2)^{\frac{1}{2}}\}^2.$$

9. If
$$P_r = \frac{\{p + (r-1) \cdot q\} \{p + (r-2) \cdot q\} \dots p}{[r]},$$

and
$$Q_r = \frac{p(p-q) \dots \{p - (r-1) \cdot q\}}{[r]},$$

prove that $P_n - P_{n-1} \cdot Q_1 + P_{n-2} \cdot Q_2 - P_{n-3} \cdot Q_3 + \&c. = 0.$

10. Find an expression for the amount of a given sum for a given time at compound interest.



ANSWERS.

I.

1. $6x$. 2. $5a+3b+11c$. 3. $10x-4y$.
4. $x^4-7x^2y^2+y^4$. 5. $a^3-b^3-c^3-d^3+2bc-2bd+2cd$.
6. $6x^5-19x^4+38x^3-51x^2+41x-35$.

II.

1. $3x^3+15x^2y-xy^3$. 2. $7a^3+5b^3-7c^3-2abc$.
3. $x^3+10xy+4y^3$; 100. 4. $10a^4+a^3b+18a^2b^2-72ab^3-27b^4$.
6. -2 . 7. $x^3-18x+81$; $9x^3-12xy+4y^3$;
 $4x^2+9y^2+z^2+12xy-4xz-6yz$; $16a^4-40a^3b^2+25b^4$.

III.

1. $x^3+y^3-x^2y^2-1$. 2. -34 . 3. $x+2y-4$. 4. $1+x^2$.
5. $m^2+n^2+p^2+r^2-2mn+2mp+2mr-2np-2nr+2pr$.
6. $m^3-3m^2n+3mn^2-n^3$;
 $x^3-y^3+z^3-3x^2y+3x^2z+3xy^2+3y^2z+3xz^2-3yz^2-6xyz$.

IV.

1. a^3+a^4+1 . 2. $x^5+4x^5+2x^4+9x^2-4x+4$.
3. $a^6-6a^5+15a^4-20a^3+15a^2-6a+1$. 4. $a^3-b^3-c^3$.
5. $2ab+3ac+6bc$. 6. $-4a^2b^2$.

V.

1. $10x - y$.
2. $12a^4 - 17a^3b - 9a^2b^2 + 13ab^3 - 63b^4$.
3. $4a^3 + 8ab + 16b^2$.
4. $m^4 + 2m^3 + 3m^2 + 2m + 1$.
5. (1) -11 .
- (2) $ac - ab - bc$.
6. (a) $(x + 13)(x + 2)$.
- (b) $(x - m)(x + n)$.
- (c) $3(mx - 3ny)(mx + 2ny)$.
- (d) $(m + p - q)(m - p + q)$.

VI.

1. $a^2 + b^2 + x^2 + y^2 + 2ab - 2ax - 2ay - 2bx - 2by + 2xy$.
2. $a^2 + ab + ac + b^2 + c^2 - bc$.
3. $a^3 + ax + y$.
4. (1) $(a^2 + b^2)(a^4 - a^2b^2 + b^4)$.
- (2) $(a + b)(a - b)(a^2 + ab + b^2)(a^2 - ab + b^2)$.
- (3) $4bx(bx + 3)(bx - 3)$.
5. -6 .
6. (1) $a - (b + c)$.
- (2) $240x + 12y$.

VII.

1. $a^5 + a^4b - ab^4 - b^5$.
2. $m^2 + 2m - 9$.
3. (1) $(a - b)(c + d)$.
- (2) $(a - 12)(a - 7)$.
- (3) $(x - 12)(x + 5)$.
- (4) $(a + m - n)(a - m + n)$.
4. $2x + 7$.
5. $14 - x$.
6. -27 .
7. $2bd - b^2 - d^2$.

VIII.

1. $ab + ac + bc$.
2. -126 .
3. $a^2 + b^2 + c^2 - ab + ac + bc$.
4. (1) $(a + 12)(a - 3)$.
- (2) $(x^2 + 11)(x^2 + 11)$.
- (3) $(y + 2x + 3x)(y - 2x - 3x)$.
5. (1) 2.
- (2) 3.
- (3) 1.
6. $36x - 12y$.

IX.

1. $x^4 - 2a^2x^2 + a^4$.
2. $x^8 - x^5y + x^2y^3 - xy^5 + y^8$.
4. The 2nd and 3rd.
5. 11.
6. 14.

X.

1. $x^4 - 9a^2x^3 + 6a^3x - a^4$.
2. $x^5 + x^5y - x^2y^3 + xy^3 + y^5$.
4. The 1st and 3rd.
5. $4\frac{1}{3}$.
6. 8 half-crowns, 4 florins, 16 pence.

XI.

1. yz .
2. $a^5 - 5a^3 + 4a$; $a^3 + a^3 - 2a$.
3. $mx^3 + ns^3 + p$.
4. (1) $3(x+1)(x-4)$.
- (2) $2x^3(x-6)(x-2)$.
- (3) $(a-b)(b-c)$.
5. (1) 7.
- (2) 5.
6. 10, 11, 12.

XII.

1. $x^4 + (a-b+c-d)x^3 + (ac-ab-ad-bc+bd-cd)x^2$
 $+ (bcd-abc+abd-acd)x + abcd$.
4. (1) $2(a+b+c+d)(a-d)$.
- (2) $(x+y-z)(x-y+z+1)$.
5. 17.

XIII.

1. $(x+3)(x-20)$; $(2a-3bc)(4a^3+6abc+9b^3c^3)$; $3(a+6b)(a-5b)$.
2. (1) 6.
- (2) 1.
- (3) £21.
4. £20.
5. abc ; xyz .
6. $x-3$.

XIV.

1. $(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$; $3a^4b(a-b)(a^2+ab+b^2)$;
 $(a-b)(a-b+1)$.
2. (1) 2.
- (2) $\frac{174}{25}$.
3. 10, 15.
4. 72 ft., 36 ft., 12 ft.
5. $2a^2(a-x)$.
6. $x+1$.

XV.

1. 6, 12, 3, 27.
2. £300.
3. $x+3$.
4. $3x-2$.
5. $\frac{2x^3+4x+1}{2x+3}$.
6. $\frac{4x+3y}{12x^3+4y^3}$.

XVI.

1. $2x+5$. 2. $2x-y$. 3. $\frac{3a^3+9a+5}{5a^3-2a^2-6a}$.
4. $\frac{7m+3}{m^2+2m+3}$. 5. $x-19$. 6. 1.

XVII.

1. $72a^4x^4y^5$. 2. $42(a^4+a^3b-ab^3-b^4)$. 3. $3x^2+4y^2$.
4. $\frac{a-1}{a+1}$. 5. $\frac{x^2-4xy-y^2}{(x^2-y^2)^2}$. 6. $\frac{1}{a+b}$.
7. $\frac{8}{(x-4)(x-3)(x+4)}$.

XVIII.

1. $5ab+4ac+3bc$. 2. x^3-ax^2+bx . 3. m^2-1 .
4. $5(x+7y)(x+5y); (m+2)(m-2)(m^2-2m+4)$
 $(m^2+2m+4); (a+b)(a+b+1)$.
5. $x-3$. 6. $\frac{6a^3b+10a^2b^2-8ab^3}{a^4-b^4}$. 7. $\frac{4x}{x+1}$.

XIX.

1. $\frac{3x-2}{x-1}$. 2. $\frac{1}{a+b}$. 3. $-\frac{24}{x+2}$.
4. $(x-2)(x+7)(x-4)(x+25)$. 5. $\frac{5}{(x-2)(x-3)(5-x)}$.
6. 5. 7. $1\frac{4}{11}$.

XX.

1. $\frac{2x^2y^2}{x^2-y^2}$. 2. $\frac{a+b-c-d}{a-b-c-d}; \frac{a-2b+1}{a+2b+1}$. 3. $x^2(x^2-1)$.
4. $x-3$. 5. $\frac{b}{a}$.

XXI.

1. $p^2q - pq^2 - p^2 + q^2 + p - q$; $y^2 - 3y + 3$.
 3. $\frac{24x+218}{(4x-5)(3x+4)(4x+3)}$. 4. (1) $x=1$. (2) $x=20$.
 6. $\frac{13}{5}$.

XXII.

1. $6a+2b-2c-2d$. 2. $\frac{ac}{(a+2b+c)(a+b+2c)}$. 3. $x+1$.
 4. $x=100$.

XXIII.

1. $3b-c$.
 2. $-x^{10} + 5x^8y - 6x^6y^2 - 5x^4y^3 + 15x^2y^4 - 5x^0y^5 - 6x^2y^3 + 5xy^4 - y^{10}$.
 3. $(x-a)(x-1)$. 4. 240. 5. $5\frac{5}{7}$ days. 6. $3(x+y)$.

XXIV.

1. x^2-3x+2 . 2. $(x+2)(x+1)(x-1)(x-4)$.
 3. $\frac{a^3b+a^2c+ab^2+ac^2+b^2c+bc^2+3abc}{abc}$.
 4. (1) $x=8$; (2) $x=3$, $y=5$. 6. 72.

XXV.

1. $-5605x+5589$.
 2. $(6x^2-5xy-6y^2)(6x^2+11x^2y-xy^2-6y^3)$.
 3. $2x^3-3x^2y+5xy^2$. 4. $1+x+x^2$.
 5. (1) $x=\pm 2$. (2) $x=4$ or $\frac{9}{19}$. (3) $x=4$, $y=4$. (4) $x=-\frac{16}{3}$.
 6. 378 and 216.

XXVI.

1. $\frac{x^3}{y^3} - \frac{y^3}{x^3}$.
 2. (a) $x=5$. (b) $x=8$ or $-\frac{4}{27}$.
 (c) $x=5$, $y=9$. (d) $x=9$ or $-\frac{55}{51}$.

3. $2ax^2 - 3a^2x + a^3$. 4. $x^2 - 3x + 5$. 5. $2(p - q)$.
6. 5 and 4.

XXVII.

2. $1 + x + x^2$. 3. $x^2 - 2x - 2$.
4. Stream 3 miles, Boat 8 miles an hour.
5. $\frac{ax + by}{ax - by}$. 6. $(5x - 1)(x + 4)(2x + 3)$.

XXVIII.

1. $(x - a)^2 - b(x - a) + b^2$. 2. $\frac{a^3 - 2a^2x - ax^2}{(a - x)(a + x)^2}$.
3. (1) $x = 41$. (2) $x = 19$. (3) $x = 7$ or 2. 4. £350, £450.
5. Prussian pound = $16\frac{1}{2}$ oz., Austrian pound = $19\frac{3}{4}$ oz., a kilogramme = $35\frac{1}{4}$ oz. 6. $a - 2$.

XXIX.

1. $a^3 + 2a^2 - a + 2$.
2. (1) $x = 3$ or -7 . (2) $x = \frac{3}{7}$ or $\frac{1}{4}$.
(3) $x = 11$, $y = 2$. (4) $x = 5$ or $\frac{1}{2}$, $y = 2$ or $\frac{1}{5}$.
3. $6d$. and $5d$. 4. A thaler = $2s. 11d.$, a franc = $9\frac{1}{2}d$,
a florin = $1s. 11\frac{1}{2}d$.

XXX.

1. 3.
2. (1) $x = \frac{5}{6}$ or $\frac{4}{5}$. (2) $x = 10$, $y = 12$.
(3) $x = 5$ or $-\frac{7}{2}$. (4) $x = -\frac{19}{3}$.
3. 860. 4. 2 feet. 5. $m - 2n + p$. 6. $4xy(x^2 - y^2)$.

XXXI.

1. $a^{\frac{7}{2}} - a^{-\frac{1}{2}}b^4$. 2. $a^{\frac{2}{3}} - a^{-\frac{2}{3}}$.
3. (1) $x = \frac{9}{20}$. (2) $x = 2$. (3) $x = 9$ or 3, $y = 3$ or 9.

4. $x=5, 8, 11 \dots$; $y=8, 16, 24 \dots$ 5. 60000.

6. $9\frac{1}{11}$. 7. (i) $\frac{13a}{a^2-x^2}$. (ii) $\frac{1}{a^2-b^2}$. (iii) $\sqrt{x+1}$.

XXXII.

1. $x^m + x^{\frac{m}{2}}y^{\frac{m}{2}} + y^m$. 2. $a^{\frac{1}{2}} + b^{-\frac{1}{2}}$.

3. (i) 7. (ii) $\frac{1}{2}$ or $-\frac{1}{3}$. (iii) $x=\frac{13}{5}$, $y=\frac{7}{5}$.

5. Sum = $\frac{2(a+b)}{a-b}$, difference = $\frac{4\sqrt{ab}}{a-b}$.

6. $\sqrt{6}$ and $2x^2-1+2x\sqrt{(x^2-1)}$.

XXXIII.

1. $x^{\frac{1}{n}} - x^{\frac{1}{n}}y^{\frac{1}{n}} + y^{\frac{1}{n}}$ and $a^{\frac{n-1}{n}} + a^{\frac{n-2}{n}}b^{\frac{1}{n}} \dots + a^{\frac{1}{n}}b^{\frac{n-2}{n}} + b^{\frac{n-1}{n}}$.

2. $\frac{x}{y} = \pm \frac{5}{3}$. 3. 736. 5. $x=48, 32, 16$; $y=19, 46, 73$.

6. $\sqrt{7} + \sqrt{13}$.

XXXIV.

2. (1) $x=5$ or $\frac{3}{35}$. (2) $x=13$, $y=12$. (3) $x=\frac{5}{2}$.

3. $\frac{3}{8}$ ths. 5. a . 6. $4-\sqrt{11}$.

7. $\frac{a}{2} + \sqrt{\left(\frac{a^2}{4} - p\right)}$, $\frac{a}{2} - \sqrt{\left(\frac{a^2}{4} - p\right)}$.

XXXV.

1. $a-\beta\sqrt{-1}$. 2. m^4+n^4 . 3. x^m .

4. 30 feet. 5. $x=3$. 6. $x^{\frac{1}{2}}-y^{\frac{1}{2}}+z^{\frac{1}{2}}$.

7. $x = -\frac{p}{2} + \frac{\sqrt{(p^2-100)}}{2}$ or $-\frac{p}{2} - \frac{\sqrt{(p^2-100)}}{2}$. If p be greater than 10, both roots are real. If p be negative and numerically greater than -10, both roots are real. If p be within the limits -10 and 10, both roots are impossible.

XXXVI.

1. $1 + \frac{a}{b} + \frac{a^2}{b^2} + \frac{a^3}{b^3} + \frac{a^4}{b^4} + \frac{a^5}{b^5}$. 2. 3 miles an hour.
4. (1) $x=1$, or -8 or $-\frac{7}{2} \pm \frac{\sqrt{161}}{2}$. (2) $x=\pm 5$, $y=\pm 1$.
- (3) $x=\pm 5\sqrt{2}$ or $\pm\sqrt{-5}$.
5. $(x+7)(x+3)(x-1)$. 6. For X 296, for Y 1488, for Z 196.
7. $(5x-3)(3x^2-7x+9)(4x^2-11x+12)$.

XXXVII.

1. $4x^{-3}+3x^{-2}+2x^{-1}+1$. 2. $x^{\frac{2}{3}}+3x^{\frac{1}{3}}-1$. 3. $46\sqrt[3]{2}$.
4. £5937. 5. 99 and 121. 7. $x^{\frac{1}{2}}+2y^{\frac{1}{2}}-x^{\frac{1}{2}}$.

XXXVIII.

1. $(m+r+n)(m+r-n)(m-r+n)(m-r-n)$.
2. x^2-y^2 . 3. $x^3-4x^2-9x+36=0$.
4. (1) $\frac{a^3+5a-14}{a+9}$. (2) b . (3) 0. 5. 30. 7. 18.

XXXIX.

1. $(a+b+c-d)(a+b-c+d)(c+d+a-b)(c+d-a+b)$.
2. $x^{2^{n-1}}-y^{2^{n-1}}$. 3. $3x-5$. 4. (1) $\frac{x^2-2x+3}{2x^2+5x-3}$.
- (2) $\frac{p+q-r}{p+r-q}$. (3) $\frac{1}{xyz}$. 5. 57 and 75.
6. $\frac{b^3}{ad}$. 7. As 3 to 2.

XL.

1. (1) $\frac{1}{abc}$. (2) $\frac{12x+4x^3-4x^5}{x^2-1}$. 2. $x-7$.
3. (1) $x=3$, $y=3$. (2) $x=\frac{a^2+4c^4}{4c^3}$. 5. 525.
6. 1 mile an hour. 7. $\frac{2(x^3-y^3)}{x^3+y^3}$.

XLI.

1. 35. 2. (1) $x=17$. (2) $x=21$. (3) $x=15, y=12$
 3. $x=\pm 4$. 5. $\frac{n^3}{4}$ and $\frac{-n^3(n-2)}{4}$. 6. 81. 7. 1920.

XLII.

1. (1) $x=\frac{94}{5}$. (2) $x=10, y=20$. (3) $x=\frac{5}{7}$ or -3 .
 2. $4\frac{1}{2}$ miles. 3. $y=5\frac{7}{9}$. 4. (1) n^2+n . (2) $\frac{3^n-1}{2}$.
 (3) $\frac{2^{n+2}+3n^2-n-4}{2}$. 5. 3. 6. $\frac{3}{5}$ of a pint from
 the first and $\frac{2}{5}$ of a pint from the second. 7. $4\sqrt{2}-3$.

XLIII.

1. $1-2x+3x^2-4x^3$. 2. 1425. 3. $3\frac{1}{2}$.
 4. 100. 5. 10.

XLIV.

1. 395660. 2. $\frac{34\sqrt{5}-18}{11}$. 3. (1) $x=\frac{7}{5}$.
 (2) $x=2, 4, y=4$. 2. (3) $x=36, y=9$. 6. 180, 360 gallons.
 7. (1) $27\frac{3}{11}$ past 5. (2) 6 o'clock.

XLV.

2. x^2-1 . 3. 57. 4. 6 and 4. 6. $\sqrt[4]{\left(\frac{3}{5}\right)}$.

XLVI.

1. 13 sov., 17 guin. 2. $3r^2+2r+7$.
 3. (1) $x=4$. (2) $x=\frac{3}{4}, y=\frac{4}{3}, x=2$. 4. 0. 5. -115
 6. -9, 27, -81, 243. 7. 8d.

XLVII.

1. (1) 646. (2) $\frac{8}{13}$. 2. $5 - \sqrt{5}$.
 3. (1) $x=0, 3$ or 4. (2) $x=0$ or 2. (3) $x=2, y=4, z=6$.
 4. 2^{n^2-2n} . 5. £2, £9, £16. 6. 1, 2, 3, 4, 5.
 7. In 7 ways.

XLVIII.

1. (1) $(7x-1)(2x-5)$. (2) $2(a-c)(1-ac)$.
 (3) $(m+n) \cdot n \cdot 2m^2$.
 2. $a^{m+n} - a^{m+n-2}b^2x^2 + 2a^{m+n-2}b^2cx^2 - a^{m+n-4}c^2x^4$.
 3. (1) a^{m-1} . (2) $\frac{2x+36x^{\frac{1}{2}}y^{\frac{3}{2}}}{x-27y}$. (3) $\frac{1}{2(x+1)^2}$.
 4. $4\frac{1}{2}$ miles per hour. 5. $\frac{3}{4}; \frac{557}{1980}$. 6. 1, $3\frac{1}{2}$, 6.....
 7. 1, 3, 9, 27, 81.

XLIX.

1. (1) $x=7$. (2) $x=\frac{5}{6}$ or $\frac{4}{5}$. (3) $x=\pm 5, y=\pm 3$.
 2. $4\frac{1}{2}$, 4, $3\frac{1}{2}$, 3, $2\frac{1}{2}$. 3. 7, 14, 28, 56, 112, 224.
 4. 25 per cent. 5. 4 gallons from first, 3 gallons from second.
 6. $\frac{6}{20}$. 7. $x=2, y=3$.

L.

1. (1) $4 - \sqrt{7}$. (2) $5 + \sqrt{6}$. (3) 4. 2. 75 and 25.
 3. (1) $x=14$. (2) $x=2, y=1$. 5. 6; 12.
 6. $(3x-7)(x-1)(5x+6)$. 7. $\frac{27}{40}$.

LI.

1. 0 and 3. 2. $x^5 - 10x^4 + 33x^2 - 36$ and $x^3 - 3$.
 3. (1) $x^2x^5(a+x)^5$. (2) $(x-15)(x+13)$.
 (3) $-(a+b+c)(a+b-c)(a-b+c)(a-b-c)$.

4. $x-1$ and $x^5+5x^4-2x^3-29x^2-5x+30$. 5. $\frac{x}{y}-2-\frac{2y}{x}$.
 6. (1) $x=17$. (2) $x=3, y=2$. (3) $x=7$ or 11 .
 7. 7 and 3. 8. 1. 9. $a-b+c+2\sqrt{ac}$.

LII.

1. 1. 2. $ac-bc-b^2+a^2$.
 3. (1) $x=4\frac{1}{2}$ (2) $x=2, y=7$.
 (3) $x=\frac{16}{3}$ or $-\frac{2}{3}$. (4) $x=\frac{7}{2}, y=\frac{2}{7}$.
 4. $(ac+bd)(bc+ad)$. 5. 20 and 30.
 6. $3x^2-2xy+5y^2$. 7. (1) -165 . (2) $\frac{3n^2+n^3}{2}$.
 8. $\frac{n(mb-na)}{n^2-m^2}, \frac{n(nb-ma)}{n^2-m^2}$. 9. $10\frac{10}{23}$ minutes past 4.

LIII.

1. $5x^7-19ax^6+35a^2x^5-52a^3x^4+39a^4x^3-26a^5x^2+14a^6x-5a^7$.
 2. $a^4-2a^3x+4a^2x^2-2ax^3+x^4$.
 3. (1) $\frac{2x^2+3x-5}{7x-5}$. (2) $\frac{-x^4+x^3-2x^2+x-1}{6(x^3+1)(x^2+1)}$.
 4. (1) 5. (2) $4\frac{1}{2}$. (3) 1, $-\frac{2}{3}$. 5. 1. 7. 6.
 8. 19. 9. 42.

LIV.

1. 10. 3. $y-2x$.
 5. (1) 5. (2) 60 or -50 . (3) $x=5, y=9$.
 6. 751, 752. 7. $x+x^{\frac{1}{2}}+1$. 9. $\frac{31(\sqrt{2}+1)}{4\sqrt{2}}$.

LV.

1. $a^2-m^2x^2+2mnax^3-n^2x^4$. 2. 5. 3. $35ab^3(a^4-b^4)$.
 4. (1) $\frac{x^3}{x^3+y^3}$. (2) $\frac{x(1+x+x^3)}{1+x^3}$.

5. (1) $x=2$ or 1. (2) $x=1$ or $\frac{1}{21}$. (3) $x=4$, $y=1$.

6. $mq : np$. 8. $\frac{a}{2b} + \frac{5b}{a} - 2$.

LVL

1. $12lmn$. 2. $x^2 - 5x + 6$. 3. $\frac{16 + 4x - 2x^2 + 13x^3 - 6x^4}{16}$.

4. (1) $x = \frac{1}{3}$ or $\frac{1}{16}$. (2) $x = \frac{5}{2}$.

(3) $x=2$ or $-\frac{2}{9}$; $y=1$ or $\frac{1}{9}$. 5. 20, 22, 24.

6. $2x^2 - 3x + 1$. 7. 20. 8. $2 + x^{-\frac{1}{2}} + x^{-\frac{1}{3}} + x^{\frac{1}{2}} + 2x^{\frac{1}{3}} + 2x^{\frac{1}{6}}$.

LVII.

2. $x^4 - 4x^2yz + 7y^2z^2$. 3. $(a^2 - x^2)(a + 2x)$.

4. (1) $\frac{(a+b-c)(a-b+c)(b+c-a)}{(a+b+c)^2}$. (2) $\frac{32x^4}{16x^4 - 1}$.

7. 3, 12, 48. 8. $x^{\frac{1}{2}}y^{-\frac{1}{2}} + 1$. 9. 13 years.

LVIII.

1. $-\frac{4}{5}$. 2. $x^3 + 6x^2 + 11x + 6$. 3. (1) $x=7$.

(2) $x=5$, $y=2$. (3) $x=1$ or $-2\frac{3}{4}$; $y=1$ or $3\frac{1}{2}$.

5. $3\frac{1}{3}$ hours. 6. (1) $7x^2 + 8x + 5$. (2) $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + x^{\frac{1}{6}}$.

7. 89 : 80. 9. $3\sqrt{5} + 5\sqrt{3}$.

LIX.

2. $x^2 - 2ax - bx + ab + a^2 + b^2$.

3. (1) $\frac{x^2 - xy + 2y^2 - x + y}{x^2 - 4y^2}$. (2) 0. 4. $3x^2 + a^2$.

6. $17\frac{1}{2}$, 35, 70 days. 7. 71; 3. 8. -1. 9. $\frac{2}{45}$.

LX.

1. $x^2 + y^2 + z^2 + 1$. 2. $x^2 - 2x + 3$.
 3. (1) $\frac{a^2 + b^2}{2ab}$. (2) $\frac{x^2 + y^2}{x + y}$. 5. $\frac{1}{3}$. 6. $-2\frac{3}{4}$.
 7. $3x^2 + 2x - 1$. 8. 28. 9. $\frac{3}{4} \left\{ 1 - \left(-\frac{1}{3} \right)^2 \right\}$.

LXI.

1. $21a - 27b + 6c$. 3. $x^2 + y^2 + z^2 + 1$. 4. 4.
 5. (1) $x = 4$. (2) $x = 4$. 6. $x - 7$.
 7. A had £13, B had £27. 9. 51.

LXII.

1. $x^3 + 8x^2y - 7xy^2 - y^3$. 2. (1) 0. (2) $\frac{1}{x-y}$.
 3. (1) $x = 3$. (2) $x = 3, y = 2$. (3) $x = 1$ or $-\frac{5}{6}$.
 4. 84 seconds. 5. $x^2 + 3x + 1$. 6. 45.
 8. $a - cx - \frac{b^2}{a}x^2 + \frac{c^2}{a}x^3$. 9. $x^{\frac{3}{2}} + 2x^{-\frac{1}{2}} + 1 + 2x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$.

LXIII.

1. $ab + cd$. 2. $(7x - 2)(4x - 1)(3x - 1)$.
 4. (1) $x = 2$. (2) $x = 3$. (3) $x = 2\frac{1}{4}$ or $-\frac{4}{3}$.
 5. $a^{\frac{2}{3}} - a^{-\frac{2}{3}}$. 6. 23. 7. 12 and 20. 8. 160.
 9. $5\sqrt{3} + \sqrt{7}$.

LXIV.

1. $x^4 + x^3 - 2x - 2$. 2. $x - 10$ and $x^3 + 10x^2 - 104x - 960$.
 4. (1) 2. (2) $\frac{2(y+z)}{x+y+z}$.
 5. (1) $x = 1\frac{1}{6}$. (2) $x = 1, y = 2$. (3) $x = 2$. 6. 324.
 8. $-a^2 - b^2 - c^2 + 2ab + 2ac + 2bc$. 9. 25 months.

LXV.

1. $x^2 + 2x + 3$.
2. $-(a^2 + b^2 + c^2 + ab + ac + bc)$.
4. (1) $x = 18$. (2) $x = -11\frac{1}{7}$. (3) $x = 5$ or $\pm\frac{1}{2}(11 + 3\sqrt{21})$.
5. 6399 and $14396\frac{15}{16}$.
6. 6.
9. $x^{\frac{4}{3}} - a^{-\frac{1}{3}}x^{\frac{1}{3}} + a^{\frac{2}{3}}$.

LXVI.

1. $a^2 + 3ab - 5b^2$.
2. (1) $x = 7$. (2) $x = \frac{a^2 - b^2}{an - bm}$, $y = \frac{a^2 - b^2}{am - bn}$.
3. 8.
4. (1) $\frac{x^2 - 8x + 15}{3x - 13}$. (2) $\frac{x^{\frac{1}{2}} + 1 - y^{\frac{1}{2}}}{x^{\frac{1}{2}} + 2 - 3y^{\frac{1}{2}}}$.
8. $x^2 - y + 2xz^{\frac{1}{2}} + z^{\frac{3}{2}}$.
9. $4\sqrt{2} + 3$.

LXVII.

1. $x^4 + px^2 - qx^3 + pq$.
2. $x^3 + x^2 + x - 6$.
3. (1) $\frac{x+2}{x^2-3x-5}$. (2) $\frac{2(a^2d^2+b^2c^2)}{abcd}$. (3) $\frac{pr^2x^2+q^2x+prs}{grs}$.
4. $x^{\frac{1}{2}} + 2x^{\frac{1}{3}} + 3x^{\frac{1}{4}} + 5$.
5. 20 and 12.
6. 8 shillings, 3 half-crowns, 9 sixpences.
7. (1) $x = 2$ or 3 or 5. (2) $x = \pm 5$ or $\pm 5\frac{1}{2}$, $y = \pm\frac{1}{2}$ or $\pm 1\frac{1}{2}$.
8. A 's 100, B 's 60.

LXVIII.

1. (1) $d(d-2c)$. (2) $-3x(x^2+1)$.
2. $4a^2 - 6ab - 8b^2$.
3. $3x-7$.
4. (1) $\frac{3x-5}{7x-11}$. (2) $\frac{3x+4}{2x^2+5x-1}$.
6. 25.
7. $x^2 + \frac{a}{x} + \frac{b}{x^2}$.

LXIX.

1. $-\frac{50}{9a}$.
2. $x-2p$.
3. (1) $\frac{a+b+c+1}{a-b-c}$. (2) $\frac{ab}{a+b}$.

4. (1) $x=a$ or $\frac{b}{a}$. (2) $x=3$ or -13 .
 (3) $x=\pm 2$. (4) $x=\pm 4$, $y=\pm 5$.
 5. 100 gallons. 6. $x^2 - \frac{xy}{\sqrt{2}} + y^2$. 8. 200. 9. $\frac{8}{9}$.

LXX.

1. (1) $\frac{(a+b+c)(a-b-c)}{(a+b)(a-b)}$. (2) $\frac{(a+c-b)(b+c-a)}{2ab}$. 2. $x^{2n}+2$.
 3. (1) $x=6$. (2) $x=5$ or 70 . (3) $x=\pm 2$, $y=\pm 1$.
 4. 2 sovereigns, 25 shillings. 8. £26. 11s. 6 $\frac{3}{4}$ d.
 9. $\sqrt{21}-2$.

LXXI.

1. x^2-2x+4 .
 2. $(x+y+a-b)(x+y-a+b)(a+b+x-y)(a+b-x+y)$.
 3. (1) $x=0$, $y=0$. (2) $x=b$, $y=a$. 4. 90 miles.
 7. $\frac{3}{4} \cdot \frac{12^{11}-1}{11 \times 12^{10}}$. 8. $-(ab)^{x+y}$ and e^{2x} .
 9. $x^4-x^3-4x^2+4x=0$.

LXXII.

1. $(x^4-a^4)(x+a)$. 2. $3x^2+ax+bx+b^2$.
 3. (1) $x=-1$. (2) $x=4\frac{1}{2}$ or $\frac{1}{2}$.
 (3) $x=\frac{(72b^3-ab)a}{18b^3-3a^2}$, $y=\frac{ab^2-12a^2b}{6b^3-a^2}$.
 4. 16 shillings and 14 shillings. 5. $\frac{1}{2}$.
 7. $\frac{57\sqrt{2}-25\sqrt{3}-9\sqrt{6}}{3}$.
 8. $2x-3(a^2-x^2)^{\frac{1}{2}}\{(a+x)^{\frac{1}{2}}-(a-x)^{\frac{1}{2}}\}$. 9. 8 years.

LXXIII.

1. $24xyz$.
 2. $x^2+x^{-2}-2(x+x^{-1})+3$ and $a^3+9a^2+21a-9-8a^{-1}+36a^{-2}-54a^{-3}$.
 3. a^2 . 4. $x=1$ or 9 , $y=9$ or 1 .

6. 20 gallons of wine and 30 of water.

7. (1) 2. (2) $15\sqrt{3}$. (3) $3+\sqrt{5}$. 8. $15\frac{1}{2}$ miles.

LXXIV.

1. (1) $(x+14)(x-13)$. (2) $(a^2-3c^2)(a^2-4b^2+c^2)$.
 (3) $(2x-1)(x+2)$.
 2. (1) $x=\frac{5}{27}$. (2) $x=2$. (3) $x=-2$ or $\frac{4}{3}$.
 3. $2x^2-3xy+4y^2$. 4. 68 half-crowns, 31 half-sovereigns.
 6. 4 hours or 7 hours. 7. $\frac{3}{5}, \frac{1}{5}, \frac{1}{15}$.
 8. $\frac{2a^2}{a^2+b^2}$. 9. $(-1)^p \cdot a^{p+1} x^{m-p-1} + a^m$.

LXXV.

1. a^2+5a+3 . 2. $x^2-5ax+7a^2$.
 3. (1) $-2\left(\frac{x^3-a^3}{x^3+a^3}\right)$. (2) 1.
 4. $x^3-7x+\frac{4}{3}$. 5. $\frac{x^2y^3+1}{y^3(x^2+y^2)}$
 6. (1) $x=\pm a, y=\pm b$. (2) $x=10, y=5$.
 (3) $x=\frac{2}{9}, y=-\frac{31}{9}$.
 7. 13 hours. 8. 265720. 9. 4 or 5.

LXXVI.

1. $3(a-b)^2$. 2. $7x^2+4xy+y^2$.
 3. (1) $x=6$ or $-5\frac{2}{3}$. (2) $x=-12\frac{3}{5}$ or 4.
 (3) $x=2a, -a, a, -\frac{a}{2}$. (4) $x=\pm 4$ or $\pm 2, y=\pm 2$ or ± 4 .
 4. 1560 feet per second. 5. (1) $2\frac{13}{16}$. (2) $10\frac{1}{8}$.
 6. (1) $\frac{x+a-b}{x-b}$. (2) x^2+1+t^2 8. 3:2.
 $4x^3-8a^2x+2=0$

LXXVII.

1. $ac - b^2$. 3. $\frac{x^2 + x - 1}{x + 2}$. 4. 1020 yards. 5. 2 3, 6, 9.
 6. (1) $x = 8$ or -5 . (2) $x = 9$ or 1, $y = 1$ or 9.
 7. $x^{\frac{1}{5}} - 2y^{\frac{1}{3}}x^{\frac{1}{3}} + 3y^{-\frac{1}{3}}x^{\frac{2}{5}} + yx^{-\frac{2}{5}} - 3y^{\frac{2}{3}}x^{\frac{2}{5}}$.
 8. $39\frac{1}{2} - 12\sqrt{10} + 2\sqrt{30} - 6\sqrt{3}$. 9. $-21\frac{3}{5}$.

LXXVIII.

1. $px^2 + qx + r$. 2. $\frac{3x - 2y}{5x + 3y}$. 3. (1) $x = 4$. (2) $x = 6$.
 6. $x^{\frac{1}{2}}y^{\frac{1}{2}} + 1$. 7. $\frac{1}{(\sqrt{x} + 1)\sqrt{(x + 1)}}$. 9. 2 hours and 3 hours

LXXIX.

1. $a^{\frac{1}{2}} + 1 + x^{-\frac{1}{2}}$. 2. $\frac{b^3}{a}$. 3. $x^2y^4x^{-3}$ and x . 4. 3 inches.
 5. $\frac{a(a^2 - 3b^2)}{a^2 - b^2}$. 7. $x^{\frac{4}{5}} - a^{-\frac{2}{5}}x^{\frac{7}{5}} + a^{\frac{4}{5}}$.

LXXX.

1. $x(a + 1) - y(b - 1)$. 2. $(x + y)^3(x - y)$. 3. $x - \frac{1}{x} - 1$.
 4. (1) $x = 5$ or 1. (2) $x = 4$ or 3.
 (3) $x = 1$ or 28, $y = 1$ or -17 .
 5. 1 quart and $1\frac{1}{2}$ quarts. 6. $a - \frac{x^2}{2a} - \frac{x^4}{8a^3} - \frac{x^6}{16a^5}$.

LXXXI.

1. (1) $\frac{1 + 2x}{1 + x}$. (2) $\frac{a + b - c}{a - b + c}$.
 2. (1) $x = 10$. (2) $x = -6$, $y = -3$. (3) $x = \pm 9$, $y = \pm 3$.
 3. $a^{\frac{3}{2}} + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b^{\frac{3}{2}} - a^{\frac{1}{2}}c^{\frac{1}{2}} - b^{\frac{1}{2}}c^{\frac{1}{2}} + c^{\frac{3}{2}}$. 5. $\frac{281}{1260}$.
 6. B . 8. A 's rate : B 's rate $= 1 : \sqrt{2} - 1$.
 9. $(2x + 3)(3x - 2)(x - 2)$.

LXXXVI.

1. $x^5 - 2a^2x^3 + a^5$. 2. $x + 3$.
3. (1) $x = 5$. (2) $x = 12$ or 6 .
- (3) $x = 2$ or $-\frac{1}{3}$, $y = 4$ or $1\frac{2}{3}$.
4. A had £72 and B had £108. 5. $x^{\frac{3}{2}} - x^{\frac{1}{2}}y^{-\frac{1}{2}} + y^{\frac{1}{2}}$.
7. 6. 8. $\frac{1}{2}(n-2)$. 9. $1 + 9x + 54x^2 + 270x^3$.

LXXXVII.

1. $\frac{7xy}{4(x^2 - y^2)}$. 2. $\frac{25\sqrt{3+43}}{2}$ and $\frac{a + \sqrt{(a^2 - x^2)}}{x}$.
3. 8 minutes. 4. $x = \frac{17 \pm \sqrt{277}}{2}$ or $\frac{17 \pm \sqrt{282}}{2}$.
5. $\sqrt{2}\sqrt{-1}(a+b\sqrt{-1})$.
9. 1.85733, 1.74062, 2.13033.

LXXXVIII.

1. $3x^3 + 7x^2 + 6x + 14$. 2. (1) $x = -b$. (2) $x = a$ or 0 .
3. $a = 20$, $b = 85$. 4. £81, 9 in the crew.
6. $m = 35$, $n = 7$. 7. 125.203303213.

LXXXIX.

1. $(a+c)(a+b)(b+c)$.
2. (1) $x = 1$ or -5 . (2) $x = 1$ or 2 , $y = -2$ or -1 .
3. $33\frac{13}{40}$ miles. 4. 6, 12, ∞ . 7. 365.2113.
8. $\frac{1 \cdot 3 \cdot 5 \dots (2r-1)}{2^r \cdot [r]} x^r$. 9. $(x+1)(3x-2)(5x-7)$.

XC.

1. 0. 2. (1) $x = \pm 1$, or 2, or 3.
 (2) $x = 2$, or $\frac{1}{2}\sqrt[3]{20}$, or $\frac{3}{2}\sqrt[3]{4}$; $y = 1$, or $\frac{1}{2}\sqrt[3]{20}$, or $\frac{1}{2}\sqrt[3]{4}$.
 4. $\frac{n \cdot (n-1) \dots (n-3r+3)}{1 \cdot 2 \dots (3r-2)} (2a)^{n-3r+1} \cdot (-3x)^{3r-1}$.
 7. 8145060. 8. 3'33984, $\bar{4}$, $\bar{1}$ 71088. 9. 2 : 5.

XCI.

2. (1) $x = 3$ or -1 . (2) $x = 9$ or -6 , $y = 6$ or -9 .
 4. 250. 5. The third. 6. 720.

XCII.

1. $x^2 - a^2 + (n+1)ax$. 2. $1 + x^{\frac{1}{2}}$.
 4. (1) $\frac{a+b}{a^2-b}$. (2) $\frac{(e^x-1)(x^2-x+1)}{(e^x+1)(x-1)}$.
 5. (1) $x = b-1$. (2) $x = 4(a-1)$.
 (3) $x = \frac{1}{2}$ or $-\frac{4}{3}$, $y = \frac{1}{3}$ or $-\frac{3}{2}$.
 7. 276 miles.
 8. $\frac{\left(\frac{1}{3}\right)^{r-1} \cdot (-2)(-5) \dots (7-3r)}{r-1} a^{\frac{1}{2}-r} \cdot (-x)^{r-1}$. 9. 46.

XCIII.

1. (1) $(ax+by)(abx+cay-efx)$. (2) $(x+13)(x-15)(x-1)$
 2. (1) $x = 5$ or $-3\frac{1}{2}$. (2) $x = \frac{a}{6}(-11 \pm \sqrt{13})$.
 (3) $x = \pm 1$ or $\frac{\sqrt{2}}{2}$, $y = \pm 2$ or $\frac{3\sqrt{2}}{2}$.
 4. The 7th. 5. $n = 10$.

XCIV.

1. Times from 1st boat passing lower end :

$$(1) \frac{v_2}{30(v_2-v_1)} \quad (2) \frac{v_2+1}{30(v_2-v_1)}$$

2. (1) $x = 3a$ or $-4a$. (2) $x = \pm 2$ or $\sqrt{\frac{-675}{26}}$.
 (3) $n = 9$. 4. $700000a^4b^4$.

XCV.

1. $\frac{2(1-R)}{5R-1}$. 2. $3 : 5$. 3. $\frac{14}{729}$.
 5. $\frac{1}{x^7} + \frac{7}{x^8} + \frac{28}{x^{11}} + \frac{84}{x^{13}}$.
 6. (1) $x = 0$ or $\frac{7}{12}$. (2) $x = 0, 1$, or $\frac{1}{2}(1 \pm \sqrt{2})$,
 $y = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ or $\pm \frac{1}{2}$.
 9. 198 days or -239 days.

XCVI.

1. (1) $2(ab-1)(b-a)$. (2) $(x-a)(x-b)(x^2-ab)$.
 3. (1) $x = a$ or b . (2) $x = 0$ or -1 . 5. 60. 6. The 8th.

XCVII.

1. $(x+1)^2 \cdot (x-2)(x-3)(3x-5)$. 2. $1+3x-6x^2+10x^3$.
 3. 6h. $38\frac{1}{4}$ m. P.M.
 4. (1) $x = \frac{64+3a^3}{16\sqrt{3}}$. (2) $x = \frac{1+\sqrt{-11}}{2}$, $y = -1$ or 2 .
 5. 19958400. 6. $\frac{10 \cdot 7 \dots (16-3r)}{3^{r-1} \cdot \sqrt[r-1]{r-1}} (a^{\frac{1}{2}})^{\frac{16-3r}{2}} (-x^{\frac{1}{2}})^{r-1}$.
 8. $\frac{2221}{1210}, \frac{31}{22}, 13$. 9. $-30\frac{5}{8}$.

XCVIII.

2. $1 + \frac{1}{3}x^3 + \frac{2}{9}x^4 + \frac{14}{81}x^5 + \frac{35}{243}x^6$. 4. (1) 5040. (2) 1440.
 5. (1) $x = -1$ or $\frac{1}{4}(-1 \pm \sqrt{5})$.

$$(2) \ x=5, 4 \text{ or } \frac{1}{2}(16 \pm \sqrt{-174}), \ y=4, 5 \text{ or } \frac{1}{2}(16 \mp \sqrt{-174}).$$

$$6. \ 5265. \quad 7. \ \frac{559130}{1594323} \left(\frac{5}{2}x^2\right)^{-\frac{2}{3}} \left(\frac{2}{5}y^2\right)^{\frac{1}{3}}.$$

XCIX.

$$2. \ 2a+3b+c.$$

$$3. \ (1) \ 2\sqrt{2}+\sqrt{3} \text{ or } 2\sqrt{2}-3\sqrt{3}.$$

$$(2) \ x=\pm\sqrt[4]{\frac{13}{3}} \text{ or } \pm 2\sqrt[4]{\frac{13}{3}}, \ y=\pm 2\sqrt[4]{\frac{13}{3}} \text{ or } \pm\sqrt[4]{\frac{13}{3}}$$

$$6. \ 900. \quad 7. \ \left(x+\frac{7r^3}{16}\right)\left(x-\frac{13r^3}{32}\right)=0.$$

$$9. \ \frac{(1-n)(1-2n)\dots\dots(1-n^2+2n)}{n^{n-1} \cdot \underline{n-1}} n^{\frac{1}{n}-n+1} \cdot \left(-\frac{x}{\sqrt{2}}\right)^{n-1}.$$

C.

$$2. \ r_1-a : r_2+a.$$

$$3. \ \frac{1 \cdot 3 \cdot 5 \dots\dots (2r-1)}{2^r \cdot \underline{r}} x^r.$$

$$4. \ 916895. \quad 6. \ n=8. \quad 7. \ (1) \ a. \ (2) \ b. \ (3) \ c.$$

$$8. \ a_1b_2-a_2b_1+a_2b-a_1b_2+ab_1-a_1b=0. \quad 9. \ 3.9798.$$

CI.

$$1. \ 10 \text{ and } 12 \text{ miles an hour.} \quad 2. \ n=7.$$

$$3. \ (1) \ x=0, 26, \text{ or } 17; \ y=0, 130 \text{ or } -68.$$

$$(2) \ x=8 \text{ or } 64, \ y=64 \text{ or } 8.$$

$$4. \ 1+6x+21x^2+56x^3, \ \frac{6 \cdot 7 \dots\dots (4+r)}{1 \cdot 2 \dots\dots (r-1)} x^{r-1}.$$

$$5. \ 9, 15, 25.$$

$$6. \ 262; \ 24.$$

$$8. \ 6720.$$

$$9. \ 5, 4, 4; \ \sqrt[3]{27}=2^{\frac{32}{3}}.$$

CII.

1. $xy(x^3 - 9y^3)$. 2. (1) $x = \frac{4a}{5}$. (2) $x = \frac{1}{4}$ or $\frac{3}{4}$ or $\frac{\sqrt{3}}{4}$; $y = \frac{3}{4}$
 or $\frac{1}{4}$ or $\frac{\sqrt{3}}{4}$. 3. $83\frac{1}{2}$ per cent. 4. $\frac{11 \cdot 14 \cdot 17}{(3)^3} (5x)^6$.
 5. $x = 2$. 8. $\cdot 371466, \bar{1} \cdot 571464$.

CIII.

1. 30 minutes. 3. $x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} - \frac{1}{4}x^{\frac{5}{2}} - \frac{1}{16}x^{\frac{7}{2}} - \frac{5}{64}x^{\frac{9}{2}} \dots$
 4. (1) $x = 0$. (2) $x = 5$ or 3 , $y = 3$ or 5 . 5. $r = \frac{n-2}{2}$.
 6. $\frac{m+n}{m+n}$. 8. 1, 3, 5.

CIV.

2. $2\frac{a^{\frac{1}{3}}}{c^{\frac{1}{3}}} - 1 + 2\frac{c^{\frac{1}{3}}}{a^{\frac{1}{3}}}$.
 3. A overtakes B $1\frac{611}{622}$ minutes after starting, and at
 $775\frac{865}{2799}$ yards, and A overtakes C $3\frac{21}{29}$ minutes after
 starting and at $1396\frac{16}{29}$ yards.
 5. The series is $\frac{2}{3}, -\frac{8}{9}x, \frac{32}{27}x^2, \dots$

$$\text{The sum } \frac{-2 \left\{ \left(-\frac{4x}{3} \right)^n - 1 \right\}}{4x + 3}.$$

8. $\frac{(2n+1)(2n)\dots(n+2)}{1 \cdot 2 \dots n} a^n x^n (a+x)$. 9. 60.

OV.

1. $(x-1)(x-2)(x+3)(x+4)$. 2. $(9+4\sqrt{3})$ miles per hour.
 3. (1) $x = 4$ or 9 , $y = 9$ or 4 .
 (2) $\left[\frac{1}{2(b+1)} \left\{ -(b+2) \pm b\sqrt{2b+3} \right\} \right]^2$.
 5. 25200. 7. $280x^4$. The fifth.

CVI.

2. $x=3, 1$, or $2 \pm \sqrt{-7}$. 3. B and E .
 5. $np \cdot (np-1) \dots (np-r+1)$

CVII.

1. $n(n-1)^2 \cdot (n-2)$. 3. $[2n-2\{(2n-3)(2n+4)\}]$
 6. (1) $x=4, -3$, or $\frac{1}{2}(1 \pm \sqrt{-43})$.
 (2) $x=\frac{9}{4}$ or $\frac{\sqrt[3]{4}-1}{\sqrt[3]{4}}$; $y=16$ or $4\sqrt[3]{4}$.
 9.
$$\frac{\left(\frac{1}{5}\right)\left(\frac{6}{5}\right)\left(\frac{11}{5}\right) \dots \left(\frac{5r-9}{5}\right)}{r-1} (a^5)^{\frac{4-5r}{5}} \cdot (-x^5)^{r-1}.$$

CVIII.

2. (1) $x=16$ or 9 . (2) $x=1, 10$ or -8 .
 (3) $\sqrt{x}=4$ or $\frac{244}{9}$, $\sqrt{y}=2$ or $\frac{242}{9}$.
 5. $\frac{a}{r-1} \left\{ \frac{r(r^n-1)}{r-1} - n \right\}$. 6. $n=\frac{p(p-q)}{p^2-q}$, $r=\frac{p-q}{p^2-q}$.
 8. $5, 6, 7, 8$.

CIX.

1. $(x-y)(y-z)(x-z)$.
 2. (1) $x=6, -7$ or $\frac{1}{2}(-1 \pm \sqrt{221})$. (2) $x=2$ or -1 .
 3. $8\frac{8}{19}$ minutes. 5. $x\left(1 - \frac{x^2}{2} - \frac{x^4}{4} - \frac{x^6}{16}\right)$.
 6. 2535. 7. (1) 15. (2) 12.
 8. $\frac{13}{2}, \frac{5}{2}; \frac{13}{2}, \frac{15}{2}$. 9. 40th day.

CX.

3.
$$-\frac{x^{\frac{1}{2}}\{(-ax^{-\frac{1}{2}})^n-1\}}{ax^{-\frac{1}{2}}+1}.$$

 4. $\frac{1}{a^3} + \frac{3x^3}{2a^5} + \frac{15x^4}{8a^7} + \frac{35x^5}{16a^9}$ and $\frac{5 \cdot 7 \cdot 9 \dots (2r-1)}{4 \cdot 5 \cdot 6 \dots (r-1)} \cdot \frac{x^{2r-2}}{a^{2r+1}} \cdot \frac{1}{2^r}.$
 5. 86400. 6. 113'1101322320. 7. 22 miles.

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